

Galois theory — Exercise sheet 4

<https://www.maths.tcd.ie/~mascotn/teaching/2019/MAU34101/index.html>

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Answers are due for Tuesday November 19th, 3PM.

Exercise 1 *Galois groups over \mathbb{Q} (100 pts)*

Prove that the following polynomials have no repeated root in \mathbb{C} , and determine their Galois group over \mathbb{Q} . *Warning: Some polynomials may be reducible!*

1. (10 pts) $F_1(x) = x^3 - 4x + 6$,
2. (10 pts) $F_2(x) = x^3 - 7x + 6$,
3. (10 pts) $F_3(x) = x^3 - 21x - 28$,
4. (10 pts) $F_4(x) = x^3 - x^2 + x - 1$,
5. (60 pts) $F_5(x) = x^5 - 6x + 3$, *using without proof the fact that this polynomial has exactly 3 real roots.*