

Galois theory — Exercise sheet 1

<https://www.maths.tcd.ie/~mascotn/teaching/2019/MAU34101/index.html>

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Answers are due for Tuesday October 1st, 3PM.

Exercise 1 *Small non-prime finite fields (50 pts)*

1. (10 pts) Make a complete list of all finite fields (up to isomorphism) with at most 30 elements and which are not isomorphic to $\mathbb{Z}/p\mathbb{Z}$ for some prime $p \in \mathbb{N}$.
2. (30 pts) Give an explicit construction for each of them.
3. (10 pts) Make a list of all pairs (K, L) such that K and L are in your list and that L contains a copy of K (up to isomorphism).

Exercise 2 *Two models for \mathbb{F}_8 (50 pts)*

Let $K = \mathbb{F}_2[x]/(x^3 + x + 1)$ and $L = \mathbb{F}_2[x]/(x^3 + x^2 + 1)$.

1. (5 pts) Prove that K and L are fields.
2. (15 pts) Determine the number of elements of K , and of L . Why does your answer imply that K and L are isomorphic?
3. (30 pts) Describe explicitly an isomorphism between K and L .

Hint: Write $L = \mathbb{F}_2[y]/(y^3 + y^2 + 1)$. Which equation does the class of $y+1 \in L$ satisfy? (Remember that $z = -z$ in characteristic 2, since $2z = 0$.)