Math 345 - Algebraic number theory
Spring 2017–2018

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Classes: Monday, Wednesday, and Friday, 12:00 – 12:50 PM

The subject of this course will be *algebraic number theory*. This can be understood in two ways, as algebra applied to number theory, but also as the study of algebraic numbers.

An *algebraic number* is a real or complex number that is a root of a polynomial with integer coefficients. For instance, $\sqrt{2}$ and $i = \sqrt{-1}$ are algebraic numbers, whereas π is not.

These numbers occur naturally when one attempts to solve *Diophantine equations*. For instance, in order to solve the equation

$$y^2 - 2 = x^3$$

in integers, it is natural to factor it into

$$(y + \sqrt{2})(y - \sqrt{2}) = x^3.$$

The introduction of the number $\sqrt{2}$, which is irrational, then raises many questions. For instance, can we say that if $y + \sqrt{2}$ and $y - \sqrt{2}$ are coprime, then the equation implies that they are both cubes ? And what does *coprime* exactly mean in this context ? This hints at factorization of integers into primes, but $\sqrt{2}$ is of course not an integer, so how can the notion of integers be generalized to include $\sqrt{2}$? Which of the usual properties of integers will subsist in this new framework ?

Prerequisites

Excellent familiarity with abstract algebra (such as the order of an element in a group, the Cayley-Hamilton theorem on the characteristic polynomial of a matrix, fields, quotient groups and quotient rings...) and basic number theory (factorization and primes, $\mathbb{Z}/n\mathbb{Z}$, ...).

Topics to be covered

A complete set of lecture notes is available at `https://staff.aub.edu.lb/~nm116/teaching/2018/math345/Poly.pdf`.

The lecture will be divided into the following chapters :

1. Number fields

   Field extensions, degrees, minimal polynomial, trace and norm, resultant.

2. Algebraic integers

   Integral elements, orders, discriminants.

3. Ideals and factorization

   (Non-)unique factorization, prime and maximal ideals, finite fields, Dedekind domains, factorization of ideals, ramification

4. The class group

   Failure of unique factorization, finiteness of the class group, application to Diophantine equations

5. Units

   Roots of unity, Dirichlet's theorem, application to the Pell-Fermat equation and to class group computations

6. Geometry of numbers (if time permits)

   Euclidian lattices, Minkowski's theorem, proof of Minkowski's bound and of Dirichlet's theorem

The lectures will focus on explicit examples and computations.

References

- Algebraic Number Theory and Fermat's Last Theorem (Ian Stewart & David Tall)
- Number fields (Daniel A. Marcus)
- A course in computational algebraic number theory (Henri Cohen), if you are interested in the algorithmic side of the subject

Homework

Exercise sheets will be handed at the end of every chapter except the one on geometry of numbers, and will have to be returned in class one week later. These exercises are mandatory and will be taken into account for the final grade.

Exam dates

The exams are tentatively scheduled as follows:

- Midterm exam: Wednesday April 11, 18:30PM–19:30PM, room to be announced later.
- Final exam: to be announced later.

Grading scheme

- Homework: 20%
- Midterm exam: 30%
- Final exam: 50%

Important: Final grades will be assigned with a curve that will not be determined before the final exam.

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