UNIVERSITY OF DUBLIN

XMA11111

TRINITY COLLEGE

Faculty of Engineering, Mathematics
and Science

school of mathematics

JF Maths
JF TP
JF TSM

Trinity Term 2012

Course MA1111 — Linear Algebra I

Dr. Paschalis Karageorgis

Attempt all questions. All questions are weighted equally.
Non-programmable calculators are permitted for this examination.

1. (a) (15 points) Compute the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}.$$

   (b) (10 points) Find a quadratic polynomial f with $f(1) = 0$, $f(2) = 3$, $f(3) = 10$.

2. A square matrix A is called lower triangular, if $a_{ij} = 0$ whenever $i < j$.

   (a) (15 points) Show that the product of two $n \times n$ lower triangular matrices is lower triangular.

   (b) (10 points) Show that the determinant of a lower triangular matrix is equal to the product of its diagonal entries.

3. (a) (15 points) Suppose that A, B are $n \times n$ matrices with $AB = 0$. Show that the column space of B is contained in the null space of A.

   (b) (10 points) Find a basis for both the null space and the column space of

$$A = \begin{bmatrix} 1 & 2 & 4 & 1 & 0 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 1 & 3 & 1 & 3 \end{bmatrix}.$$

4. (a) (10 points) Let $T\colon V \to W$ be a linear transformation between vector spaces and let $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ be vectors in V. Suppose $T(\boldsymbol{v}_1), T(\boldsymbol{v}_2), \ldots, T(\boldsymbol{v}_n)$ are linearly independent. Does it follow that $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ are linearly independent? Explain.

   (b) (15 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ denote the reflection along the line $y = x$. Find the matrix of this linear transformation with respect to the basis

$$B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}.$$