TUTORIAL 1

MA1111: LINEAR ALGEBRA I, MICHAELMAS 2016

i). In class, we mentioned that the cross product is not associative. That is, we don't always have $u \times (v \times w) = (u \times v) \times w$. Instead, the cross product satisfies an important identity known as the *Jacobi identity*:

$$u \times (v \times w) + v \times (w \times u) + w \times (u \times v) = 0. \tag{1}$$

Show, using the identity

$$u \times (v \times w) = (u \cdot w) \cdot v - (u \cdot v) \cdot w$$

which we showed in class, that (1) holds for any vectors $u, v, w \in \mathbb{R}^3$.

ii). Prove that the diagonals of a square are orthogonal.

iii). Consider the three planes given by

$$x + 2y + z = 5, \qquad 2x + 2y + 2z = 4, \qquad x + z = -1.$$

Using row reduction, find the intersection between all three planes.

iv). Show that if $ad - bc \neq 0$, then the reduced row echelon form of the matrix $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ is $\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$.

Advanced Problem (optional):

For any three numbers x, y, z define the three vectors

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad v_3 = \begin{pmatrix} x^2 \\ y^2 \\ z^2 \end{pmatrix}.$$

For which choices of x, y, z is the set of all linear combinations of the three vectors, $\text{span}(v_1, v_2, v_3)$, equal to all of $\mathbb{R}^3$?