HOMEWORK 8

MA1111: LINEAR ALGEBRA I, MICHAELMAS 2016

Solutions are due at the beginning of class on **Thursday, December 1**. Please write your name and course on your assignment, and make sure to staple your papers.

(1) Recalling that $\mathcal{P}_{\leq 2}$ is the space of polynomials of degree at most 2 with real coefficients, consider the linear transformation $T\colon \mathcal{P}_{\leq 2} \to \mathbb{R}^2$ given by
$$T(P(x)) = (P(0), P(1)).$$
For example, we have $T(x^2+1) = (1,2)$.
  (a) Find the matrix associated to T in terms of the standard bases $\{1, x, x^2\}$ of $\mathcal{P}_{\leq 2}$ and $\{e_1, e_2\}$ of $\mathbb{R}^2$.
  (b) Find a basis for the kernel of the matrix you determined in part a). The elements of this basis will be coordinate vectors of a basis for the kernel of the linear transformation T. Write this basis for the kernel of T.

(2) Find the matrix representation of the linear map T in Problem 1 in terms of the non-standard bases $\{1-x, x, -2+x^2\}$ of $\mathcal{P}_{\leq 2}$ and $\{(1,2),(3,4)\}$ of $\mathbb{R}^2$.

(3) Consider the linear transformation $f\colon M_{2\times 2} \to M_{2\times 2}$ given by
$$f(A) = A + A^T,$$
where A^T is the transpose of A.
  (a) Consider the standard basis
$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$
of $M_{2\times 2}$. Find the matrix representing f with respect to this basis.
  (b) Find a basis for the kernel of the matrix you found in part a). The set of matrices with the vectors in this basis being their coordinate vectors is a basis for the kernel of f. Write down this basis.

(4) Suppose that $g(x) = 3 + x$. Consider the linear transformation $T\colon \mathcal{P}_{\leq 2} \to \mathcal{P}_{\leq 2}$ given by
$$T(P(x)) = P'(x) \cdot g(x) + 2P(x),$$
were $'$ denotes the derivative d/dx. Now consider the linear transformation $U\colon \mathcal{P}_{\leq 2} \to \mathbb{R}^3$ given by
$$U(a + bx + cx^2) = (a+b, c, a-b).$$
  (a) In terms of the standard basis $\{1, x, x^2\}$ of $\mathcal{P}_{\leq 2}$, compute the matrix representing T.

(b) In terms of the standard basis of $\mathcal{P}_{\leq 2}$ and the standard basis $\{e_1, e_2, e_3\}$ of $\mathbb{R}^3$, compute the matrix representing U.

(c) Using the matrices you found in a) and b), find the matrix representing the composition UT by taking the product of these two matrices.

(d) Directly compute the matrix representing the composition UT using the same bases as above, and check that the result is the same as the matrix product you took in part c).