HOMEWORK 4

MA1111: LINEAR ALGEBRA I, MICHAELMAS 2016

Solutions are due in class on **Thursday, October 27**. Please put your name and student number, as well as your subject (Maths., TP, or TSM) on the back of your assignment, and make sure to staple your papers.

(1) Find the determinant of the matrix

$$A = \begin{pmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{pmatrix}.$$

(2) Use the method of adjoints to compute the inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 7 & 0 & 9 \end{pmatrix}.$$

(3) A matrix A is called *upper triangular* if all entries below the main diagonal are 0, i.e., if $A_{ij} = 0$ whenever $j < i$. For example,

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 6 & 7 & 7 \\ 0 & 0 & 11 & 12 \\ 0 & 0 & 0 & 16 \end{pmatrix}$$

is upper triangular. Show that the determinant of any $n \times n$ upper triangular matrix A is the product of its entries lying on the diagonal, i.e., $\det A = A_{11}A_{22}\cdots A_{nn}$. **Hint:** Consider a row expansion along the bottom row of A. What do you observe?

(4) According to our definition in class, B is an inverse for A if $AB = BA = I_n$. Suppose we instead require only that $AB = I_n$. In general algebraic contexts, this will not be enough to guarantee that B is an inverse for A. However, there is enough extra structure in the theory of matrices to conclude in this situation that B is an inverse for A. This problem will guide you through the proof of this fact.

   (a) Show that if A, B are square matrices of size $n \times n$ with $AB = I_n$, then A is invertible. (Hint: Use determinants).

   (b) Using the notation and results of part (a), show that in fact $B = A^{-1}$.