UNIVERSITY OF DUBLIN

TRINITY COLLEGE

Faculty of Engineering, Mathematics
and Science

school of mathematics

JF Maths/TP/TSM **Michaelmas Term 2016**

MA1111 — Linear Algebra I

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Attempt all questions. All questions are weighted equally.
No calculators are permitted for this examination. All work must be shown for full credit.

1. Consider the system of linear equations

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ -2x_1 - 3x_2 + x_3 = -1 \\ 2x_1 + x_2 - 3x_3 = 1. \end{cases}$$

   (a) Write down an equivalent equation $Ax = b$, and find A^{-1}.

   (b) Use your answer from part (a) to solve the original system of linear equations.

2. The real polynomials in one variable x of degree at most 2 are denoted by $\mathcal{P}_{\leq 2}(\mathbb{R})$.

   (a) Prove that the subset $\{1+x, 2+x+x^2, 4-3x+x^2\}$ is a basis of $\mathcal{P}_{\leq 2}(\mathbb{R})$.

   (b) Find the coordinate vector of $1+x+2x^2$ with respect to this basis.

3. Let $f\colon M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ be the linear transformation defined by $f(A) = BA - A^T$, where $M_{2\times 2}(\mathbb{R})$ is the space of 2×2 real-entry matrices, and $B = \left(\begin{smallmatrix} 1 & 2 \\ 0 & 1 \end{smallmatrix}\right)$.

   (a) Find the matrix associated to f with respect to the standard basis

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}.$$

   (b) Use your answer from (a) to find a basis for the image of f.

   (c) Use your answer from (a) to find a basis for the kernel of f.

4. (a) Find the determinant of $A = \left(\begin{smallmatrix} -2 & 0 & 0 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{smallmatrix}\right)$ **directly from the definition we gave in class.**

   (b) Find the determinant of A^{-1}.

5. (a) Given a subspace W of $\mathbb{R}^n$, prove that the *orthogonal complement* $W^\perp$, defined by $W^\perp = \{v \in \mathbb{R}^n | v \cdot w = 0 \text{ for all } w \in W\}$ is a subspace of $\mathbb{R}^n$.

   (b) Prove that if W is a subspace of $\mathbb{R}^3$, then we have the direct sum decomposition $\mathbb{R}^3 = W \oplus W^\perp$.

6. Let A be an $n \times n$ matrix satisfying $A^2 = 2A$. Find the possible values of $\det A$.