TUTORIAL 9

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) Evaluate $\int_0^1 \int_x^{2x} \int_{xy-1}^{x+y} x^2 dz dy dx$.

(2) Use spherical coordinates to evaluate $\iiint_R x dV$ when R is the piece lying in the first octant of the unit ball $x^2 + y^2 + z^2 \leq 1$ centered at the origin. (Recall that $\sin^2 \vartheta = \frac{1-\cos(2\vartheta)}{2}$.)

(3) Use a change of variables to find $\iint_R (y^2 - x^2)^4 dA$ where R is the trapezoid with vertices at $(0,1)$, $(1,0)$, $(2,0)$, and $(0,2)$. (Hint: Make a change of variables which transforms two of the sides of the trapezoid to be on lines of the form $u = a$ and $u = b$, and to find a suitable second parameter v, make a choice which makes the integrand as nice as possible.)

Advanced Problem:

Consider the n-variable function $e^{-\frac{1}{2}\left(x_1^2+x_2^2+\ldots x_n^2\right)}$. Express $\int \cdots \int_{\mathbb{R}^n} e^{-\frac{1}{2}\left(x_1^2+x_2^2+\ldots x_n^2\right)} dV$, where dV is an n-dimensional volume element, as a product of integrals to find its value. Additionally use the very important technique of differentiating under the integration sign, which states that for "nice" functions $f(x,t)$ we have

$$\frac{d}{dx} \int_a^b f(x,t) dx = \int_a^b \frac{\partial f}{\partial x}(x,t) dt,$$

to evaluate the integral

$$\int_{-\infty}^{\infty} x^n e^{-x^2}$$

for any positive integer n. (Hint: take the Gaussian integral identity $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ and insert a parameter t.)