TUTORIAL 4 SOLUTIONS

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) Compute the following partial derivatives and values of partial derivatives.

(a) The partial derivatives f_x and f_y when $f(x,y) = x^4y - \sqrt{xy} + \log(x)\sin(y)$.

(b) The value

$$\frac{\partial z}{\partial x}\Big|_{x=3,\ y=2}$$

for $z = \frac{x^2+y^2}{x-y}$.

Solution: a). We find that

$$f_x = 4x^3 - \frac{1}{2}\sqrt{\frac{y}{x}} + \frac{\sin y}{x},$$

$$f_y = x^4 - \frac{1}{2}\sqrt{\frac{x}{y}} + \log x \cos y.$$

b). We first compute (using the quotient rule) that

$$\frac{\partial z}{\partial x} = \frac{2x(x-y) - (x^2+y^2)}{(x-y)^2} = \frac{x^2 - 2xy - y^2}{(x-y)^2}.$$

Plugging in at the point $(x,y) = (3,2)$, we find

$$\frac{\partial z}{\partial x}\Big|_{x=3,\ y=2} = \frac{9-12-4}{1^2} = -7.$$

(2) (a) Find the linearization, or the linear approximation, $L(x,y)$ of the function $f(x,y) = xe^{xy}$ near the point $(x_0,y_0) = (1,0)$.

(b) Use your answer from a) to approximate the value $f(0.99, 0.2)$.

Solution:

a). In general, the linear approximation function $L(x,y)$ is given by

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0).$$

In this case, $f_x = e^{xy} + xye^{xy}$, $f_x(1,0) = 1$, $f_y = x^2e^{xy}$, $f_y(1,0) = 1$, and $f(1,0) = 1$. Thus, the lineariztion is

$$L(x,y) = 1 + (x-1) + y.$$

b). The approximation given by the linearization is

$$L(x,y) = 1 + (0.99-1) + 0.2 = 1 - 0.01 + 0.2 = 1.19.$$

(The real answer, if you plugged it into a calculator, would be about 1.21, which is about a 1% relative error).

(3) We saw that Clairaut's Theorem guarantees that for "nice" functions, we can compute mixed second order partial derivatives in different orders and obtain the same answers. Here you will check a special case of this by direct computation. Namely, compute the partial derivatives directly to check that

$$f_{xz} = f_{zx}$$

when $f(x, y, z) = \sin(x + y)(x^3y - y^2z)$.

Solution: We find:

$$f_x = \cos(x + y)(x^3y - y^2z) + 3x^2y\sin(x + y)$$
$$f_{xz} = -y^2\cos(x + y),$$
$$f_z = -y^2 sin(x + y),$$
$$f_{zx} = -y^2\cos(x + y) = f_{xz}.$$