

## HOMEWORK 5

MA1132: ADVANCED CALCULUS, HILARY 2017

- (1) Consider the function  $z = f(x, y) = x \log(xy) - \sqrt{x^2 + y^2}$  with  $x = t^2 + 1$ ,  $y = t - 1$ . Find  $\frac{dz}{dt}$  by using the chain rule.
- (2) Suppose that  $w = f(x, y, z) = xy^{\frac{1}{2}} + \sin\left(\frac{x}{y}\right) \tan z - z^2 x^3$  and  $x = 2r + s$ ,  $y = st$ ,  $z = r - t$ . Find  $\frac{\partial w}{\partial r}$ .
- (3) Find  $\left. \frac{\partial^2 f}{\partial \vartheta^2} \right|_{\vartheta=\frac{\pi}{2}, r=\sqrt{3}}$  for  $f(x, y) = xy + y^2$ ,  $x = r \cos \vartheta$ ,  $y = r \sin \vartheta$ .
- (4) Find the directional derivative of  $f(x, y, z) = \frac{x+y^2}{x-y^3z}$  in the direction of the line in the plane  $z = 0$  which makes an angle of  $\pi/3$  with the  $x$ -axis (in the direction of increasing  $x$ ) as well as in the direction of the vector  $(1, 2, 3)$  at the point  $(1, -1, 1)$ .
- (5) Find a unit vector pointing in the direction in which  $f$  increases the fastest at the point  $(1, 1)$ , when

$$f(x, y) = \frac{x}{y} - \frac{y^{\frac{3}{2}}}{x}.$$

How fast is  $f$  increasing in this direction?