HOMEWORK 10

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) Find the volume between the cone $z = r$ and the plane $z = 0$ and lying under the plane $z = 10$.

(2) The *centroid* of a region in the plane is the center of mass in the case when the density function is a constant (which, since we divide out by the mass in our formula for center of mass anyways, can be assumed to be 1). Show that the centroid of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is at

$$\frac{1}{3}(x_1 + x_2 + x_3, y_1 + y_2 + y_3).$$

(3) Evaluate the integral $\iiint_R xyz dV$ where R is the part of the ball $\rho \leq 1$ lying in the first octant (i.e., when $x, y, z \geq 0$).

(4) Change variables to compute

$$\iint_R xy dA,$$

where R is the parallelogram with vertices at $(0, \pm 1)$, $(\pm 2, 0)$ by turning the integration region into a rectangle with sides parallel to the u and v axes for some coordinates u, v.

(5) Find the area of an elliptical region given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$ by finding a suitable change of variables which transforms the problem into a problem of integrating over a circular region.