# UNIVERSITY OF DUBLIN

XMA11111

## TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

**JF Mathematics** **Trinity Term 2011**
**JF Theoretical Physics**
**JF Two Subject Mod**

COURSE MA1111 — LINEAR ALGEBRA I

Friday, May 13 Exam Hall 14:00 — 16:00

Dr. Vladimir Dotsenko

ATTEMPT ALL QUESTIONS.

For each task, the number of points you can get for a complete solution of that task is printed next to it.

You may use all statements proved in class and in home assignments; when using some statement, you should formulate it clearly, e.g. "in class, we proved that if $A$ is invertible, then the reduced row echelon form of $A$ is the identity matrix".

Unless otherwise specified, all vector spaces are over the complex numbers.

Non-programmable calculators are permitted for this examination.

1. (25 points) Using elementary row operations, compute the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}$.

   Find a polynomial $f(t)$ of degree at most $2$ for which $f(1) = 1$, $f(3) = 0$, $f(4) = 11$.

2. (25 points) Write down the definition of an even permutation. For each of the permutations $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 2 & 7 & 6 & 5 & 1 \end{pmatrix}$, $\begin{pmatrix} 4 & 8 & 2 & 3 & 7 & 6 & 5 & 1 \\ 5 & 2 & 8 & 4 & 6 & 1 & 3 & 7 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 2 & 5 & 6 & 7 & 3 & 1 \end{pmatrix}$, determine whether it is odd or even.

3. (a) (10 points) Prove that for two square matrices $A$ and $B$ of the same size we always have $\mathrm{tr}(AB) = \mathrm{tr}(BA)$.

   (b) (15 points) How many *distinct* numbers can there be among the six traces

   $$\mathrm{tr}(ABC), \mathrm{tr}(ACB), \mathrm{tr}(BCA), \mathrm{tr}(BAC), \mathrm{tr}(CBA), \mathrm{tr}(CAB)$$

   for square matrices $A$, $B$, $C$ of the same size? For each variant of the answer, give an example.

4. (25 points) For the matrix

   $$A = \begin{bmatrix} 1 & 1 & 2 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 2 \\ 2 & 1 & 1 & 1 & 1 & 2 & 5 \\ -1 & 2 & 1 & 1 & 0 & 1 & 1 \end{bmatrix},$$

   compute the dimension and find a basis of the solution space to the system of equations $Ax = 0$.