UNIVERSITY OF DUBLIN

XMA12121

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Mathematics
JF Theoretical Physics

Trinity Term 2010

MA1111/MA1212 - LINEAR ALGEBRA I/II

Thursday, April 29 RDS - MAIN 09:30 — 12:30

Dr. Vladimir Dotsenko

For each task, the number of points you can get for a complete solution of that task is printed next to it.

You may use all statements proved in class and in home assignments; when using some statement, you should formulate it clearly, e.g. "in class, we proved that if A is invertible, then the reduced row echelon form of A is the identity matrix".

Unless otherwise specified, all vector spaces are over the complex numbers.

Non-programmable calculators are permitted for this examination.

1. (10 points) Denote by A the matrix $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & -2 \\ 1 & -2 & 3 \end{bmatrix}$ and by b the vector $\begin{bmatrix} 3 \\ -1 \\ 7 \end{bmatrix}$. Compute the inverse A^{-1} using elementary row operations, and use it to solve the system $Ax = b$.

2. (10 points) Is the permutation $\begin{pmatrix} 1 & 4 & 3 & 5 & 10 & 8 & 9 & 6 & 7 & 2 & 11 \\ 5 & 4 & 10 & 9 & 7 & 8 & 2 & 6 & 11 & 1 & 3 \end{pmatrix}$ even or odd? Why? Same question for the permutation $\begin{pmatrix} 1 & 4 & 3 & 5 & 10 & 8 & 9 & 6 & 7 & 2 & 11 \\ 10 & 4 & 9 & 7 & 2 & 8 & 11 & 6 & 1 & 3 & 5 \end{pmatrix}$.

3. (a) (10 points) Under what conditions is a set of vectors of a vector space V called complete? linearly independent? a basis? Prove that if a set of vectors is complete, then it remains complete after being extended by an arbitrary vector v from V.

   (b) (8 points) Assume that the set of vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ (all belonging to the same vector space V) is complete. Prove that the set of vectors $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{w}$, $2\mathbf{v}+\mathbf{w}$ is also complete. What are the possible values of $\dim V$ in this situation? Explain your answer.

4. (12 points) A linear operator $A\colon \mathbb{R}^3 \to \mathbb{R}^4$ satisfies the following conditions:

$$A(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \quad A(\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad A(\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}) = \begin{bmatrix} 2 \\ 3 \\ 1 \\ -1 \end{bmatrix}.$$

Compute $A(\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix})$.