MA2215 Fields, rings and modules

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Last year, you met several algebraic structures: groups, fields, vector spaces and sets of matrices. In this course we'll start by studying rings, which come about when you consider addition and multiplication (but not division) from an abstract point of view. If we throw division into the mix, then we get the definition of a field. We'll look at how one field can be extended to get a larger field, and use this theory to solve some geometric problems that perplexed the Greeks and remained unsolved for 2,000 years. We'll also talk about modules over a ring, which generalise the idea of a vector space over a field.

Syllabus

1. Rings; examples, including polynomial rings and matrix rings. Subrings, homomorphisms, ideals, quotients and the isomorphism theorems.

2. Integral domains, unique factorisation domains, principal ideal domains, Euclidean domains. Gauss' lemma and Eisenstein's criterion.

3. Fields, the field of quotients, field extensions, the tower law, ruler and compass constructions, construction of finite fields.

4. Modules and examples. Direct sum decompositions and applications.

Recommended books

- Peter J. Cameron, *Introduction to Algebra* (second edition) covers practically all of the course.

- John R. Durbin, *Modern Algebra, An Introduction* covers everything except modules.

Lectures, homework and tutorials We have three weekly lectures, on Mondays at 2pm in East End Lecture Theatre 3 and on Thursdays at 12pm and 4pm in the Synge Lecture Theatre. There will be no printed course notes, so you should take notes during the lectures.

I'll hand out exercises during the Monday lecture in weeks 2, 4, 6, 9 and 11. These should be handed in the following Monday.

In weeks 3, 5, 7, 10 and 12, there will be a tutorial/problem solving session instead of the Thursday 4pm lecture. We'll discuss your homework and try other problems.

Exam and grading There will be a two-hour exam at the end of the year. Your overall grade will be the larger of your final exam result, and the sum of 20% of your homework grade with 80% of your final exam result. So if H is your homework grade and E is your final exam result, then your overall grade will be $\max(E, \frac{1}{5}H + \frac{4}{5}E)$.