A reminder of some useful group theory

Let $(G, +)$ be an abelian group and let H be a subgroup of G.

1. If $x \in G$ then the right coset of H by x is the set
$$H + x = \{h + x \colon h \in H\}.$$
2. When are two of these cosets equal? Answer:
$$H + x = H + y \iff x - y \in H.$$
3. We write G/H for the collection of all of these right cosets:
$$G/H = \{H + x \colon x \in G\}.$$
4. Since G is abelian, H is a *normal* subgroup of G. We define an operation on G/H, which we usually also write as $+$, by
$$(H + x) + (H + y) = H + (x + y)$$
for all $x, y \in G$. This operation is well-defined, and $(G/H, +)$ is a group which we call the *quotient group* of G by H.
5. Let's see how the fundamental homomorphism theorem looks when we apply it to abelian groups.

   Let $\theta \colon (G_1, +) \to (G_2, +)$ be a group homomorphism, and let $K = \ker\theta$. Then

   - $\theta(G_1)$ is a subgroup of G_2,
   - $K = \ker\theta$ is a (normal) subgroup of G_1, and
   - the quotient group G_1/K is (group)-isomorphic to G_1/K via the well-defined (group) isomorphism
$$\phi \colon G_1/K \to \theta(G_1), \quad \phi(K + x) = \theta(x).$$