

Geometry and Topology, or How Different a Mug and a Doughnut Really Are

Victoria LEBED, Research Fellow in Maths





What is mathematics?

Is it this?



1

What is mathematics?

And this?

Frank and Ernest

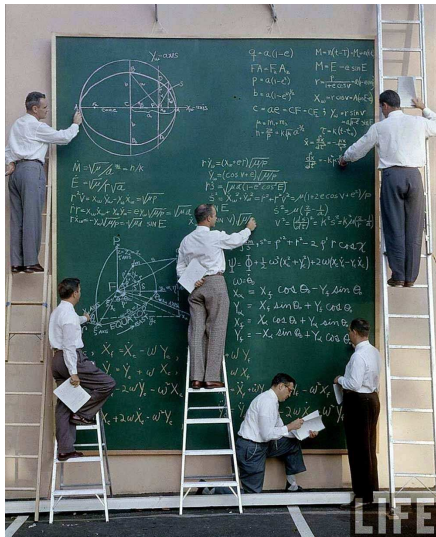


Copyright (c) 1994 by Thaves. Distributed from www.thecomics.com.

1

What is mathematics?

Sometimes yes:



NASA scientists with their board of calculations, 1961, *Life* magazine.

1

What is mathematics?

Sometimes yes:

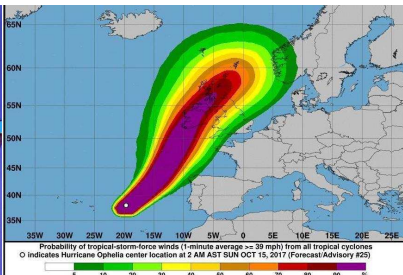


A screenshot from *Hidden Figures*.

1

What is mathematics?

But maths is also all of this:



Maths is about understanding **mechanisms** and seeing **patterns**, not only about computing! It involves **creativity**, and is sometimes closer to arts and philosophy than to sciences.

Maths is about understanding **mechanisms** and seeing **patterns**, not only about computing! It involves **creativity**, and is sometimes closer to arts and philosophy than to sciences.

Fortune 500's Most Valued Characteristics in an Employee:

Characteristics	1999	1970
Teamwork	1	10
Problem Solving	2	12
Interpersonal Skills	3	13
Oral Communication	4	4
Listening Skills	5	5
Personal Career Development	6	6
Creative Thinking	7	7
Leadership	8	8
Goal Setting/Motivation	9	9
Writing	10	1
Organizational Effectiveness	11	11
Computational Skills	12	2
Reading Skills	13	3

2

Mugs and doughnuts

Geometry

- ✓ Distances, angles, curvature.
- ✓ “Metallic mathematics”.

Topology

- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.

2

Mugs and doughnuts

Geometry

- ✓ Distances, angles, curvature.
- ✓ “Metallic mathematics”.
- ✓ mug \neq doughnut

Topology

- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.
- ✓ mug = doughnut

2

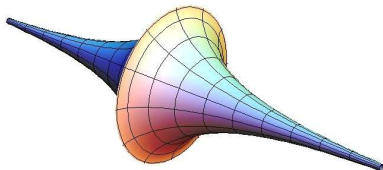
Mugs and doughnuts

Geometry

- ✓ Distances, angles, curvature.
- ✓ “Metallic mathematics”.
- ✓ mug \neq doughnut
- ✓ Examples of applications:
 - 1) surveying;
 - 2) construction;
 - 3) astronomy;
 - 4) the shape of the universe.

Topology

- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.
- ✓ mug = doughnut

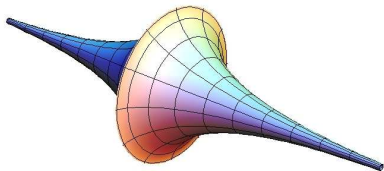


2

Mugs and doughnuts

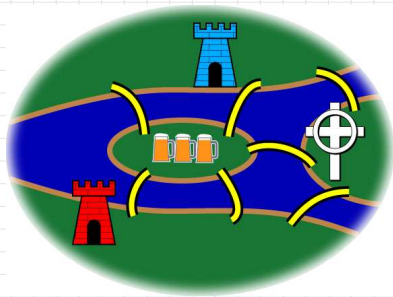
Geometry

- ✓ Distances, angles, curvature.
- ✓ “Metallic mathematics”.
- ✓ mug \neq doughnut
- ✓ Examples of applications:
 - 1) surveying;
 - 2) construction;
 - 3) astronomy;
 - 4) the shape of the universe.



Topology

- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.
- ✓ mug = doughnut
- ✓ Examples of applications:
 - 1) Seven Bridges of Königsberg Problem;

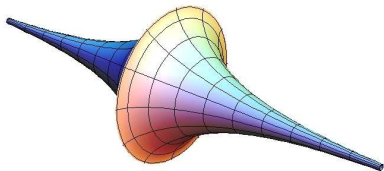


2

Mugs and doughnuts

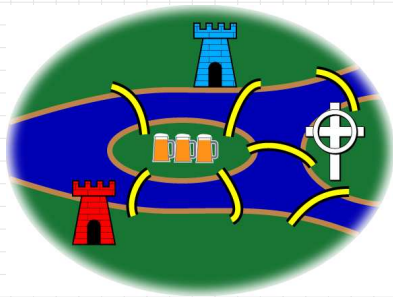
Geometry

- ✓ Distances, angles, curvature.
- ✓ “Metallic mathematics”.
- ✓ mug \neq doughnut
- ✓ Examples of applications:
 - 1) surveying;
 - 2) construction;
 - 3) astronomy;
 - 4) the shape of the universe.



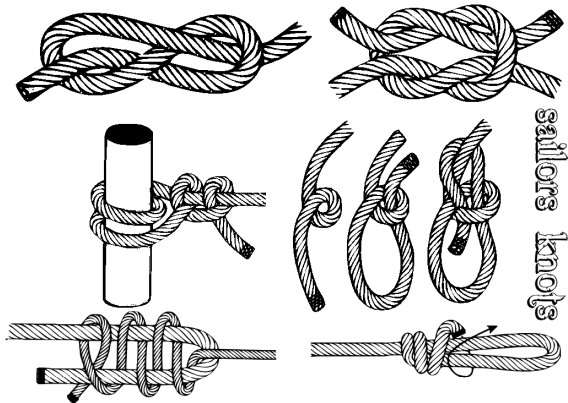
Topology

- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.
- ✓ mug = doughnut
- ✓ Examples of applications:
 - 1) Seven Bridges of Königsberg Problem;
 - 2) knot theory.



Knots and braids surround us:

- ✓ rope knots in sailing, mountaineering



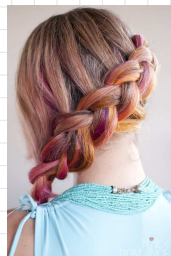
3

Knots and braids

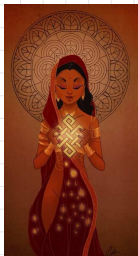
✓ ties



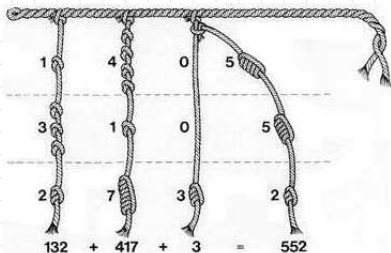
✓ hairstyles

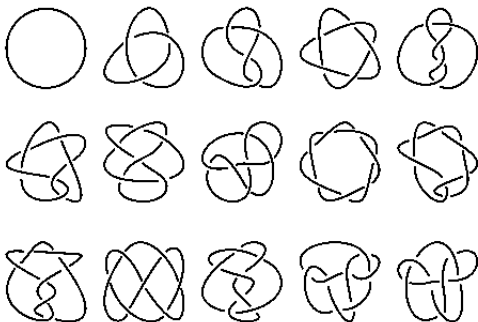


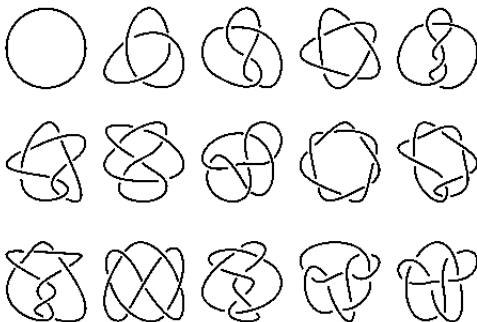
✓ decoration, religion



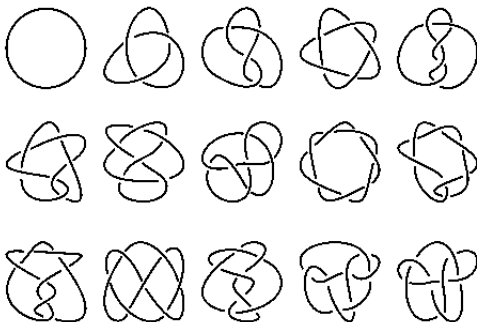
✓ quipu: used by Incas
for recording information







nature	many objects: different material, size, usage
mathematics	one abstraction: shape

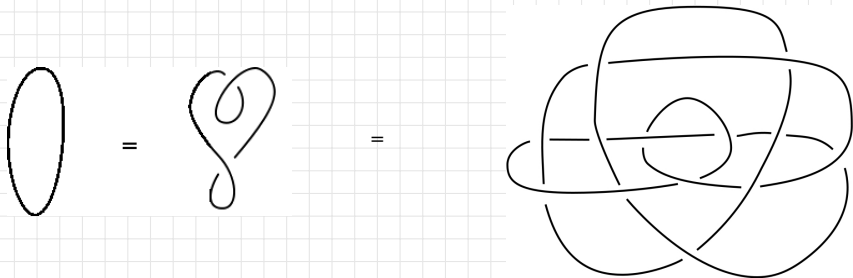


nature	many objects: different material, size, usage
mathematics	one abstraction: shape

Mathematics begins when many objects are replaced with one abstraction.

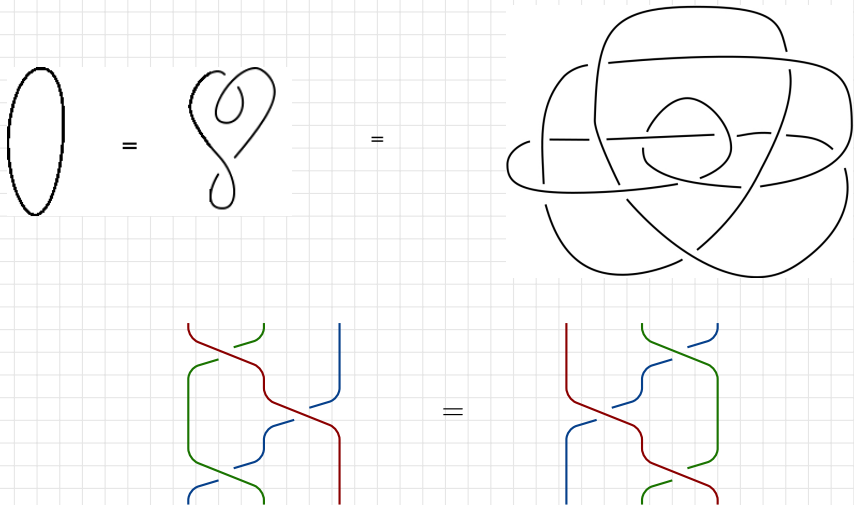
Knots and braids in mathematics

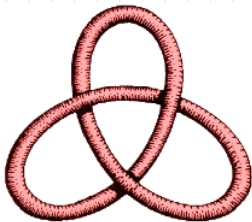
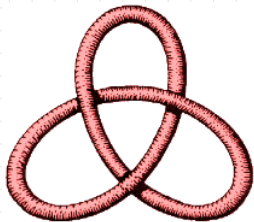
In mathematics, knots and braids are considered up to deformation:

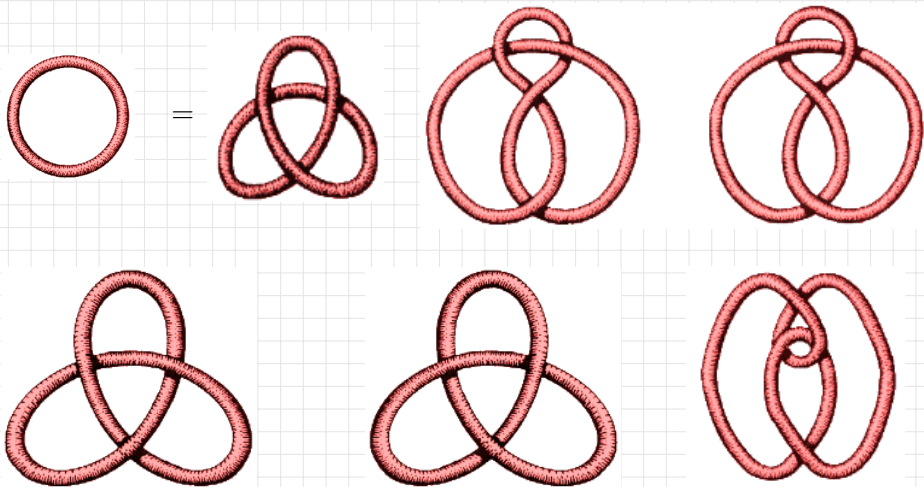


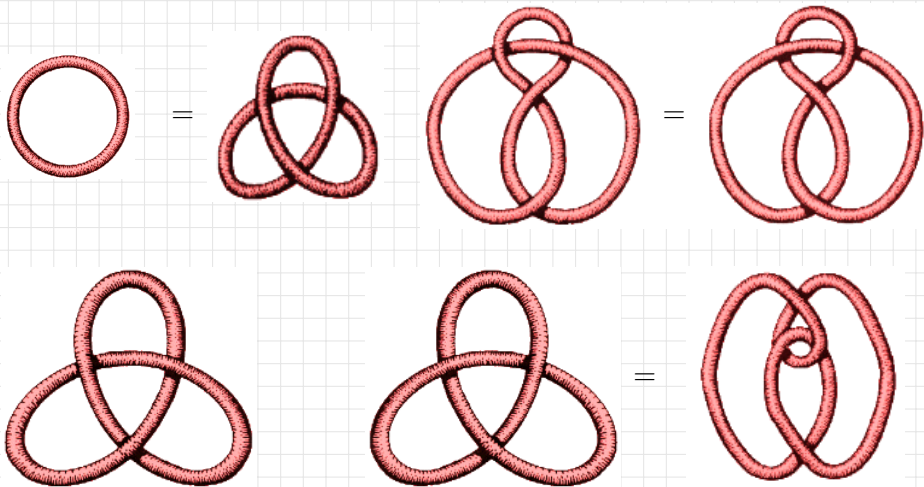
Knots and braids in mathematics

In mathematics, knots and braids are considered up to deformation:



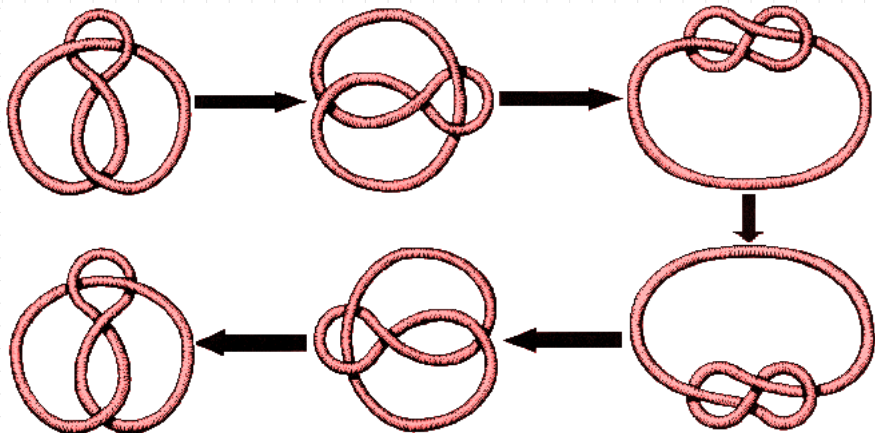






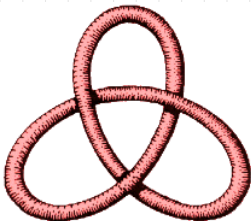
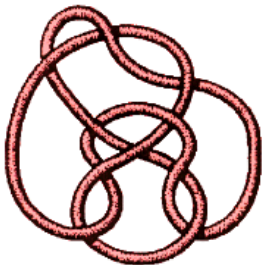
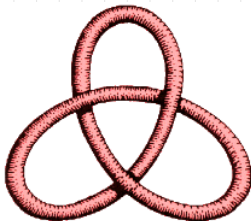
5

How to distinguish knots?



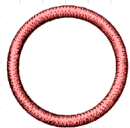
How to distinguish knots?

It is a difficult problem:

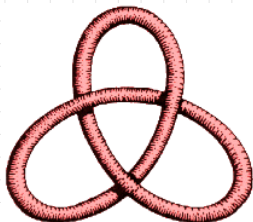
 \neq  $=$ 



Knot invariants



?
 \neq



?
 \neq

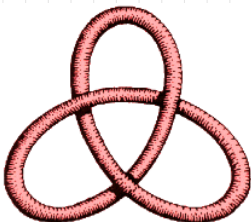




Knot invariants



?
 \neq



?
 \neq



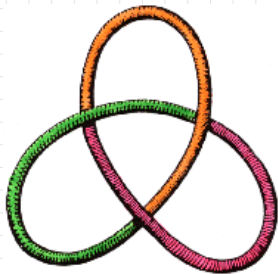
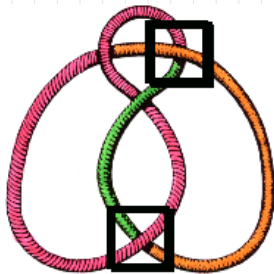
Knot colourings: 3 colours



Rule: If a knot K' is a deformation of a knot K , and K can be coloured by 3 colours, then K' can be coloured by 3 colours as well.

6

Knot invariants

 \neq  \neq 

Can be coloured ?

no

yes

no



Knot theory: history

1867: Peter Tait experimented with **smoke rings**;



Knot theory: history

1867: Peter Tait experimented with **smoke rings**;



Lord Kelvin (Thomson): atoms = knotted tubes of ether



C (Carbon)



O (Oxygen)



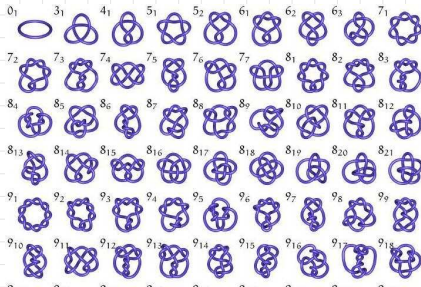
H (Hydrogen)

7

Knot theory: history

PERIODIC TABLE OF THE ELEMENTS																		VIIIA			
IA												IIIA		IVA	VA	VIA	VIIA	VIII			
H 1.00794												B 10.811		C 12.011	N 14.007	O 15.999	F 18.998	He 4.002602			
Li 6.941		Be 9.0122												Al 26.982		Si 28.086	P 30.974	S 32.06	Cl 35.45	Ne 20.180	
Na 22.990		Mg 24.305												Ga 69.723		Ge 72.64	As 74.922	Se 78.96	Br 79.904	Ar 39.948	
K 39.098		Ca 40.078		Sc 44.956	Ti 47.88	V 50.942	Cr 51.996	Mn 54.938	Fe 55.845	Co 58.933	Ni 58.693	Cu 63.546	Zn 65.39	Ga 69.723	Ge 72.64	As 74.922	Se 78.96	Br 79.904	Kr 83.80		
Rb 85.468		Sr 87.62		Y 88.906	Zr 91.224	Nb 92.906	Mo 95.94	Tc 98.906	Ru 101.07	Rh 101.07	Pd 106.32	Ag 107.87	Cd 112.41	In 114.82	Sn 118.71	Sb 121.76	Te 127.60	I 126.91	Xe 131.29		
Cs 132.91		Ba 137.33		La-Lu 138.91, 140.12, 140.91, 144.24, 145, 146.36, 151.96, 157.25, 158.93, 162.50, 164.93, 167.26, 168.93, 173.04, 174.97		Hf 178.49	Ta 180.95	W 183.84	Re 186.21	Os 190.23	Ir 192.22	Pt 195.08	Au 196.97	Hg 200.59	Tl 204.38	Pb 207.2	Bi 208.98	Po [209]	At [210]	Rn [222]	
Fr [223]		Ra [226]		89-103 Ac-Lr [227], 227.04, 231.04, 238.03, 237, 244, 243, 247, 251, 252, 257, 259, 262																	
57		58	59	60	61	62	63	64	65	66	67	68	69	70	71						
La		Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu						
138.91		140.12	140.91	144.24	[145]	150.36	151.96	157.25	158.93	162.50	164.93	167.26	168.93	173.04	174.97						
89		90	91	92	93	94	95	96	97	98	99	100	101	102	103						
Ac		Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr						
227		232	231	238	237	244	243	247	247	251	252	257	259	259	262						

?

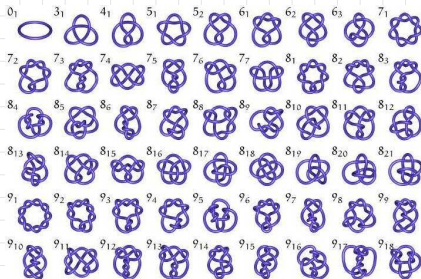


7

Knot theory: history

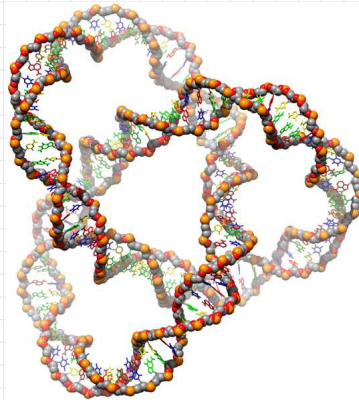
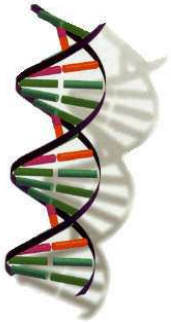
PERIODIC TABLE OF THE ELEMENTS																		VIIIA		
IA												IIIA		IVA	VA	VIA	VIIA	VIII		
H (1.0079)												B (10.811)		C (12.011)	N (14.007)	O (15.999)	F (18.998)	He (4.0026)		
Li (6.941)		Be (9.0122)											Al (26.982)		Si (28.086)	P (30.974)	S (32.06)	Cl (35.45)	Ne (20.180)	
Na (22.990)		Mg (24.305)											Ga (69.723)		Ge (72.64)	As (74.922)	Se (78.96)	Br (79.904)	Ar (39.948)	
K (39.098)		Ca (40.078)	Sc (44.956)	Ti (47.88)	V (50.942)	Cr (51.996)	Mn (54.938)	Fe (55.845)	Co (58.933)	Ni (58.693)	Cu (63.546)	Zn (65.39)	Ga (69.723)	Ge (72.64)	As (74.922)	Se (78.96)	Br (79.904)	Kr (83.80)		
Rb (85.468)		Sr (87.62)	Y (88.906)	Zr (91.224)	Nb (92.906)	Mo (95.94)	Tc (98.906)	Ru (101.07)	Rh (101.07)	Pd (106.32)	Ag (107.87)	Cd (112.41)	In (114.82)	Sn (118.71)	Sb (121.76)	Te (127.60)	I (126.91)	Xe (131.29)		
Cs (132.91)		Ba (137.33)	La-Lu (138.91-175.05)		Hf (178.49)	Ta (180.95)	W (183.84)	Re (186.21)	Os (190.23)	Ir (192.22)	Pt (195.08)	Au (196.97)	Hg (200.59)	Tl (204.38)	Pb (207.2)	Bi (208.98)	Po (209)	At (210)		
Fr (223)		Ra (226)	Ac-Lr (227-261)		Rf (261)	Db (262)	Sg (266)	Bh (264)	Hs (277)	Mt (268)	Uun (285)	Uuu (288)	Uub (285)	Uuq (289)						
57 La (138.91)		58 Ce (140.12)	59 Pr (140.91)	60 Nd (144.24)	61 Pm (145)	62 Sm (150.36)	63 Eu (151.96)	64 Gd (157.25)	65 Tb (158.93)	66 Dy (162.50)	67 Ho (164.93)	68 Er (167.26)	69 Tm (168.93)	70 Yb (173.04)	71 Lu (174.97)					
89 Ac (227)		90 Th (232.04)	91 Pa (231.04)	92 U (238.03)	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (262)					

?

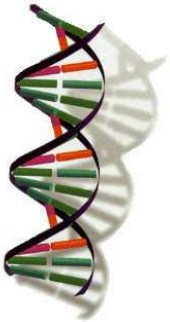


This is false!!!

✓ **Biology:** DNA molecules.

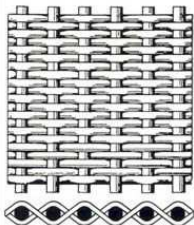
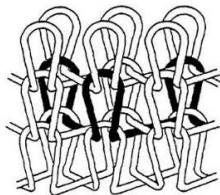
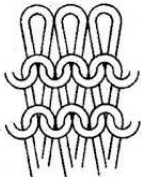


✓ **Biology:** DNA molecules.

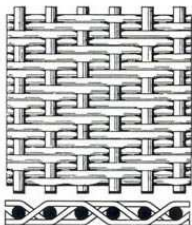


Some enzymes can cut, twist, and reconnect the DNA.
This changes the properties of the DNA.
Knot theory is used to detect the action of enzymes.

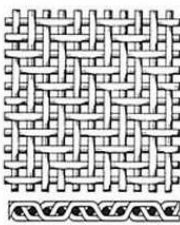
✓ **Textile:** weaving patterns.



PLAIN DUTCH WEAVE

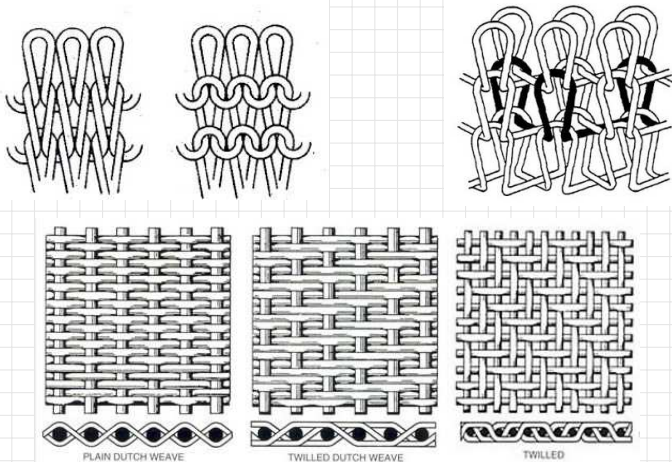


TWILLED DUTCH WEAVE



TWILLED

✓ **Textile:** weaving patterns.



✓ **The shape of the universe** question: all 3-dimensional spaces can be encoded by knots.