Lecture 1: Introduction to module MA1S11

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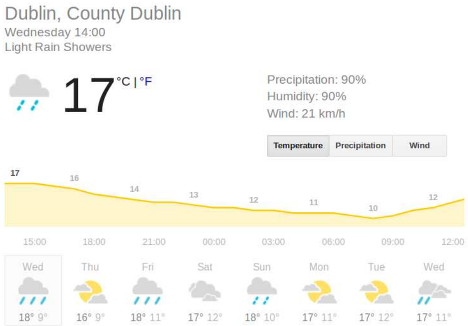
MA1S11A: Calculus with Applications for Scientists

October 2, 2017



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⚠ Exam problems $\approx$ homework problems.

So, do your homework $\Longrightarrow$ succeed in the exam!!!

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⚠ Your questions will help me make the lectures more understandable for everyone!

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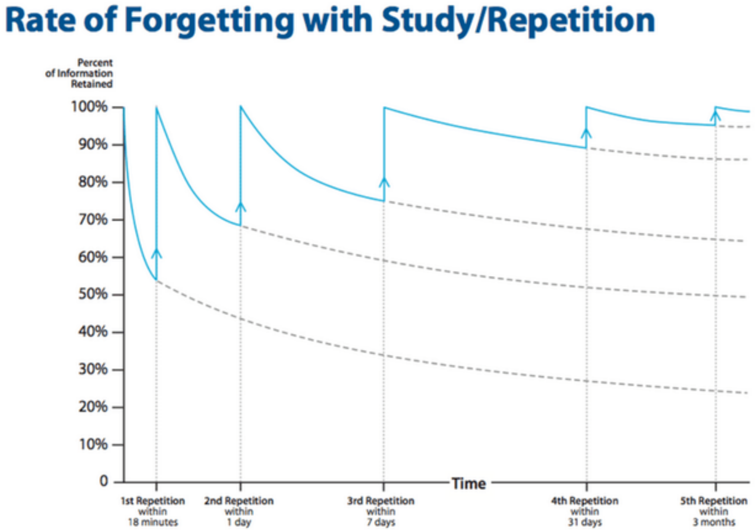
✓ After each lecture, ask yourself what it was about.

✓ In maths, everything is to be **proved**. So, prepare yourself for a (small) dose of proofs!

✓ **Active learning**: take notes, do homework and additional exercises (books & my web page).



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Fortune 500's Most Valued Characteristics in an Employee:

Characteristics	1999	1970
Teamwork	1	10
Problem Solving	2	12
Interpersonal Skills	3	13
Oral Communication	4	4
Listening Skills	5	5
Personal Career Development	6	6
Creative Thinking	7	7
Leadership	8	8
Goal Setting/Motivation	9	9
Writing	10	1
Organizational Effectiveness	11	11
Computational Skills	12	2
Reading Skills	13	3

3 What is this module about?

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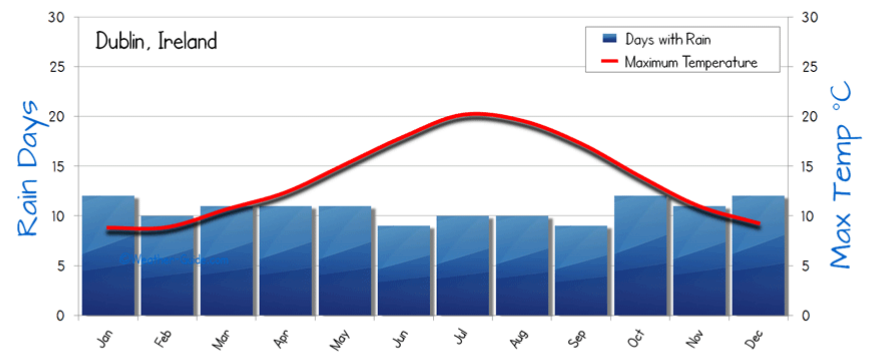
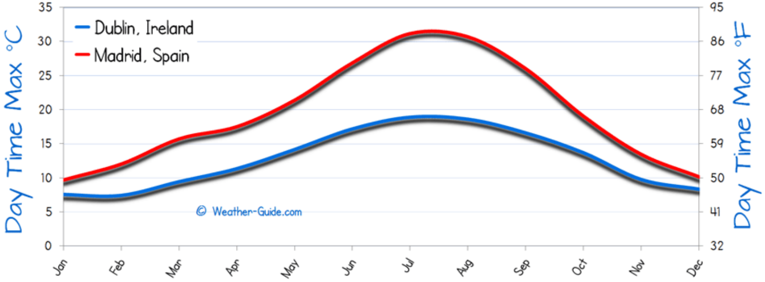
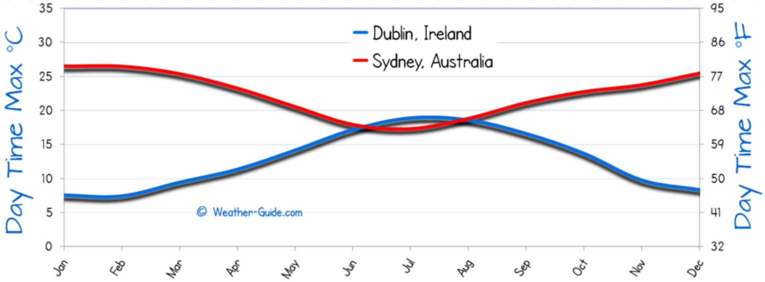
About (nice) functions!



We'll study functions,

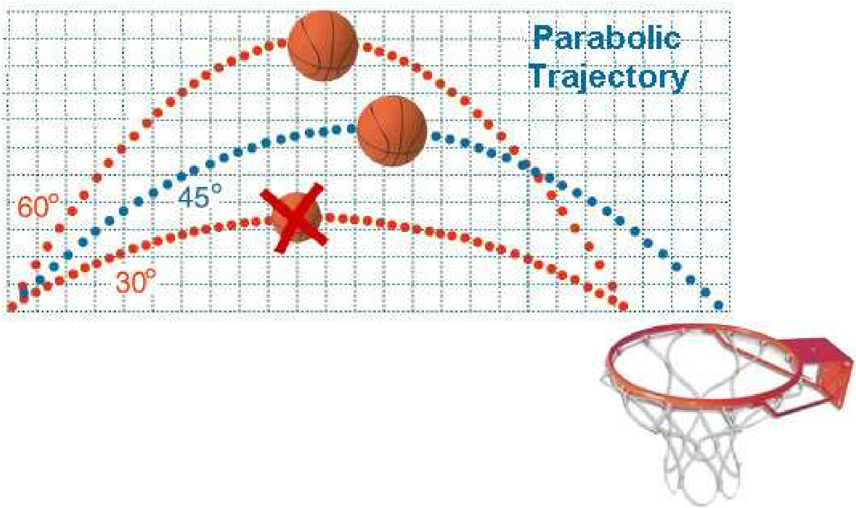




compare functions,

interpret functions.

Functions are omnipresent in science and everyday life.



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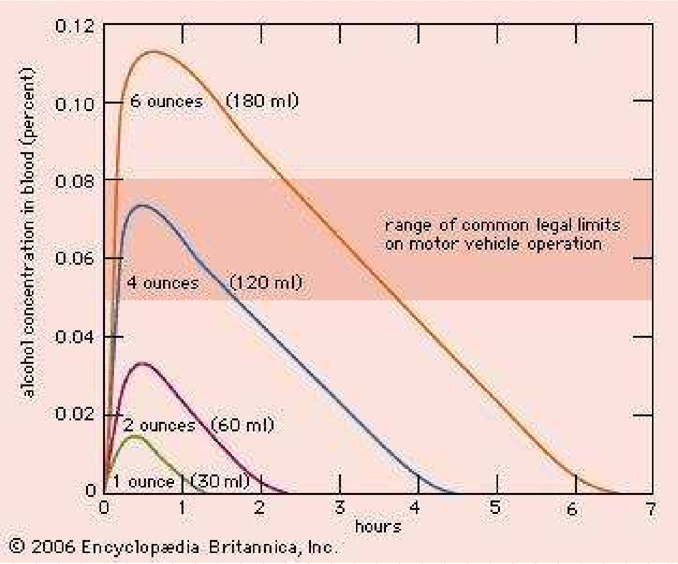

Typical questions about functions:

✓ When should I have sold my pounds? (extrema)

✓ How did the exchange rate evolve? (derivative)

✓ What is the average rate for the period? (integral)

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Definition. A (real) variable y is said to be a **function** of a (real) variable x if its dependence on x is such that for every a there is exactly one value of y corresponding to the value $x = a$.

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x	1	1.5	2	2.5	3	3.5	4
y	0	1.25	3	5.25	8	11.25	15

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A function can be represented by an algebraic formula, for example

$$y = x^2 - 1.$$

This will be our main method.

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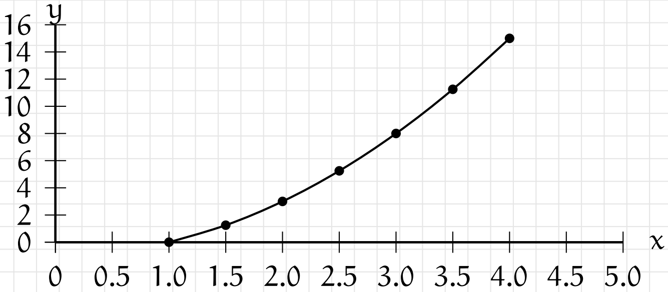

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Question. Did you notice that the four presentations described the same function?

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⚠ We can use different names for the independent variable. So, $x \mapsto f(x)$ and $t \mapsto f(t)$ denote exactly the same function f.

5 Reading functions

If a function is represented by a table, figuring out what corresponds to what is straightforward:

x	1	2	3	4	5	6	7
y	4	5	0	1	3	4	-1

5 Reading functions

If a function is represented by a formula, it is a matter of a computation to figure out what corresponds to what:

$$f(x) = x^2 - 2x + 3$$

means that

$$\begin{aligned} f(0) &= 0^2 - 2 \cdot 0 + 3 = 3, \\ f(1) &= 1^2 - 2 \cdot 1 + 3 = 2, \\ f(-2.5) &= (-2.5)^2 - 2 \cdot (-2.5) + 3 = 14.25, \\ f(\sqrt{3}) &= (\sqrt{3})^2 - 2\sqrt{3} + 3 = 6 - 2\sqrt{3}. \end{aligned}$$

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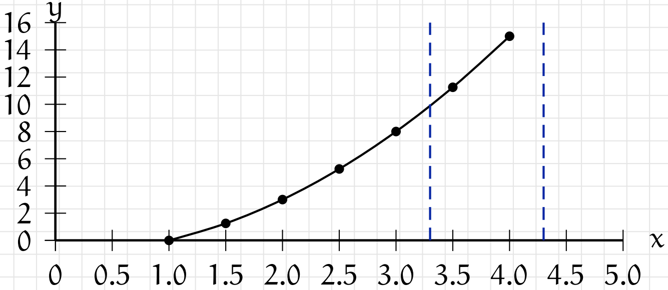
If a function is represented by a graph, to figure out what corresponds to a particular value of x, one draws a vertical line intersecting the x axis at that value, and looks for where it intersects the graph:



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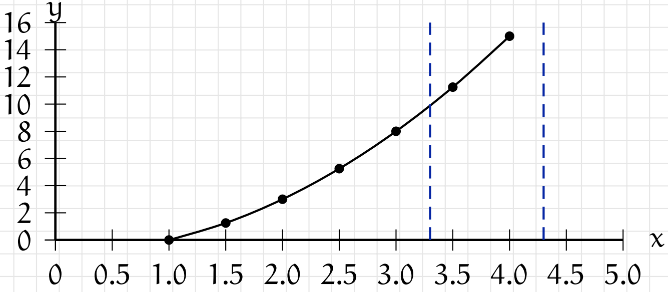

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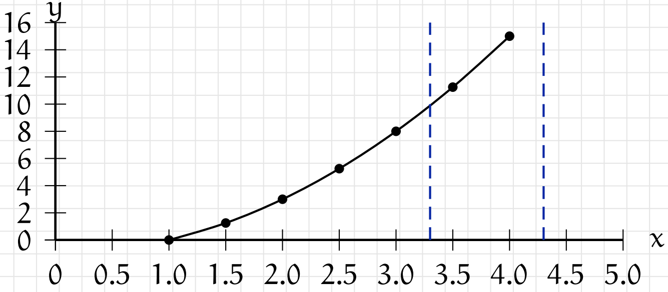

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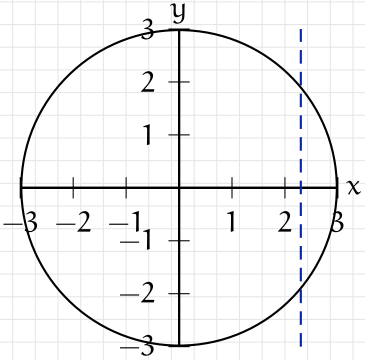
Vertical line test: A curve in the xy-plane is the graph of some function if and only if no vertical line can meet it in more than one point.

6 Vertical line test: an example

Points (x, y) for which $x^2 + y^2 = 9$ form a circle of radius 3, and there are many vertical lines that intersect that circle twice:



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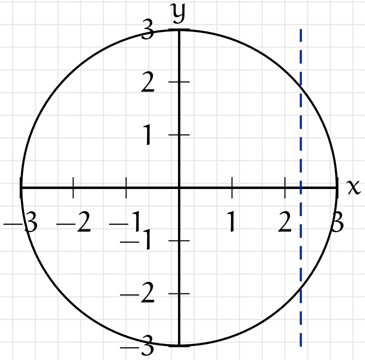

Therefore that circle is not a graph of a function.
Algebraically, $y^2 = 9 - x^2$ means that $y = \pm\sqrt{9 - x^2}$, so the value of y can only be determined up to a sign, and is in no way unique.

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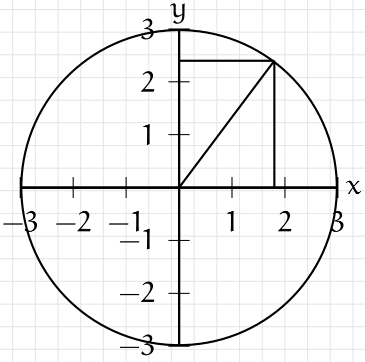
Because $x^2 + y^2$ is the distance from (x, y) to $(0, 0)$ by the Pythagoras Theorem:



7 Examples of functions: Square root

Recall that every positive real number x has two **square roots**:

- ✓ a positive one, denoted by $\sqrt{x}$;
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⚠ Students often write $\sqrt{t^2} = t$, which is false in general!

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The **absolute value** or **magnitude** of a real number x is defined by

$$|x| = \begin{cases} x, x \geqslant 0, \\ -x, x < 0. \end{cases}$$

In other words, it assigns to a number x that same number x if it is nonnegative, and strips away the minus sign if it is negative. For example, $|3| = 3$, $|-3.5| = 3.5$, $|0| = 0$.

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Important properties of absolute values are

$$|-a| = |a|,$$

$$|ab| = |a||b|,$$

$$|a/b| = |a|/|b|, \quad b \neq 0,$$

$$|a+b| \leqslant |a| + |b| \quad (\text{"triangle inequality"}),$$

$$\sqrt{a^2} = |a|.$$

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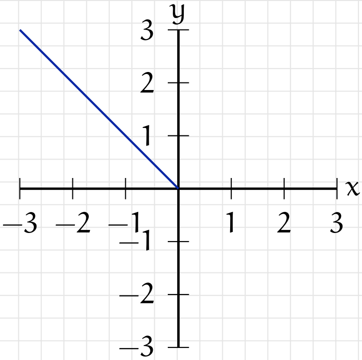

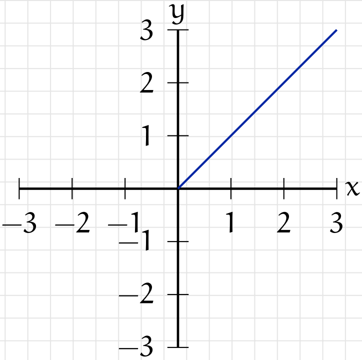

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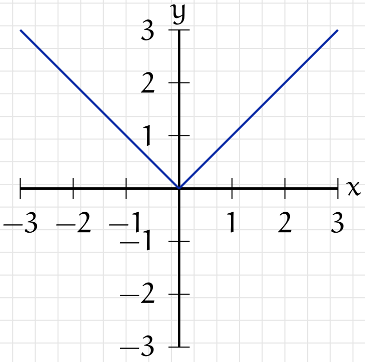

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$[a, +\infty)$ stands for the closed ray consisting of all x for which $a \leqslant x$,
$(-\infty, a)$ stands for the open ray consisting of all x for which $x < a$,
$(-\infty, +\infty)$ is a symbol notation for the whole real line $\mathbb{R}$.

9 Piecewise defined functions

As an example, consider the function

$$f(x) = \begin{cases} 1, & x \leqslant -1, \\ x^2, & -1 < x \leqslant 1, \\ 0, & x > 1. \end{cases}$$

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1) If they do agree, just plot the graph as you usually do.

2) If they do not, distinguish between open and closed endpoints. Use

- ✓ an empty circle ($\circ$) to denote the value corresponding to an open endpoint (which is an omitted point; you cut it out from the graph, rather than adding it);
- ✓ a full circle ($\bullet$) to denote the value corresponding to a closed endpoint (which is a part of your graph).

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In our example, $x = -1$ is a point of the first type; at this value, the function is continuous. The point $x = 1$ is of the second type; at this value, the function makes a jump: