

Homework/Tutorial 8

What this homework is about

You will practice in analysing functions with the help of differential calculus.

Reminder

Algorithm for graphing a rational function $f(x) = P(x)/Q(x)$

1. Check if f is given in the **reduced form** (i.e., the polynomials P and Q have no common factors). If not, find its reduced form.
2. Determine if the graph has **symmetries** about the y -axis / the origin, i.e., whether f is **even** / **odd**.
3. Find where and how the graph **meets the x -axis**, i.e., compute the **roots** of f and their multiplicities. (A **root** of f is a root c of P . It is of **multiplicity** m if $(x - c)^m$ divides $P(x)$ but $(x - c)^{m+1}$ does not.)
4. Find where the graph **meets the y -axis**, i.e., compute $f(0)$.
5. Determine all **vertical asymptotes** and check if there is a sign change across them, i.e., compute the **poles** of f and their multiplicities. (A **pole** of f is a root of Q .)
6. Describe the **behaviour of f at $\pm\infty$** : compute $\lim_{x \rightarrow \pm\infty} f(x)$, and find the **curvilinear asymptote** of the graph. (For this you need to divide P by Q , and use this to present f as $S(x) + \frac{R(x)}{Q(x)}$ with $\deg R < \deg Q$.)
7. Find the **sign of f** on each interval between the x -intercepts and the vertical asymptotes.
8. Determine where f is **increasing/decreasing**, and find all **critical points**, and **local and global extrema**. For this, analyse the **sign of f'** (if it exists).
9. Determine where f is **concave up/down**, and find all **inflection points**. For this, analyse the **sign of f''** (if it exists).
10. Sketch the graph of f .

Question

Analyse the following rational function using the plan above, and sketch its graph:

$$f(x) = \frac{x^4 - 2x^3 + x^2}{x^2 - 2x}.$$