Homework/Tutorial 4

Please hand in your work at the end of the tutorial. Make sure you put your name and student ID number on what you hand in. Please write your work in an intelligible way!

A complete solution to question 1 is worth 3 marks, to question 4 is worth 1 mark, and for the remaining questions it is 2 marks.

What this homework is about

In exercises 1-3 you will continue working with inverse functions, in particular with inverse trigonometric functions. In exercises 4-5 you will start computing limits.

Reminder

Trigonometric functions are periodic, and so do not have inverses, but their restrictions to certain intervals do. Here is a summary on classical inverse trigonometric functions:

function	domain	range	symmetries
arcsin	$[-1,1]$	$[-\frac{\pi}{2},\frac{\pi}{2}]$	odd
arccos	$[-1,1]$	$[0,\pi]$	–
arctan	$\mathbb{R}$	$(-\frac{\pi}{2},\frac{\pi}{2})$	odd
arccot	$\mathbb{R}$	$(0,\pi)$	–

A function f **has the limit** L **at the point** a if the values $f(x)$ get as close as we wish to L when x gets sufficiently close to a. This is written as $\lim\limits_{x\to a} f(x) = L$, or $f(x) \underset{x\to a}{\to} L$.

If the condition above is true only for $x > a$ (or $x < a$), one talks about **one-sided limits**, and writes $x \to a^+$ (or $x \to a^-$) in the notations above. Two- and one-sided limits are related as follows:

$$\lim_{x\to a} f(x) = L \qquad \Longleftrightarrow \qquad \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L.$$

Questions

1. (a) Determine all real c and d for which the function
$$f(x) = x^2 + cx + d, \qquad x \geq 0$$
admits an inverse.
   (b) Write down this inverse when it exists, and find its domain and range.
   (c) Plot the graph of the inverse function for $c = 2$ and $d = 2$.
   (d) When d varies, what happens to the graphs of f and f^{-1}?
2. Compute
   (a) $\tan(\arctan(3))$;
   (b) $\sin(\arcsin(-2))$;
   (c) $\arccos(\cos(-\frac{\pi}{3}))$;
   (d) $\arctan(\tan(\frac{5\pi}{8}))$.
3. Determine the natural domain of the function
$$f(x) = \arccos(x^2 - 3x + 3) - \frac{x}{\arctan(x - \frac{\pi}{2})}.$$

4. Compute $\lim_{x \to -2} \frac{x^2-4}{x+2}$.
5. The *floor* $\lfloor x \rfloor$ of a real number x is defined as the greatest integer that is less than or equal to x. For instance, $\lfloor 0.7 \rfloor = 0$, $\lfloor \pi \rfloor = 3$, $\lfloor -0.7 \rfloor = -1$, $\lfloor -4 \rfloor = -4$.
   (a) Plot the graph of the function $f(x) = \lfloor x \rfloor$.
   (b) Compute its one-sided and two-sided limits at $x = 1$, if they exist.