Homework/Tutorial 2

Please hand in your work at the end of the tutorial. Make sure you put your name and student ID number on what you hand in. Please write your work in an intelligible way!

A complete solution to every question is worth 2 marks.

What this homework is about

You'll practise in finding the natural domain and the range of functions.

Reminder

The **natural domain** of a function f defined by a formula consists of all values of x for which $f(x)$ has a well defined real value. To find it, you might use the following algorithm:

1. Whenever you see an expression of type $\sqrt{g(x)}$ (or $\sqrt[2n]{g(x)}$ with $n \in \mathbb{N}$), determine all x for which $g(x) \geq 0$.
2. Whenever you see an expression of type $\frac{h(x)}{g(x)}$, determine all x for which $g(x) \neq 0$. Beware of nested fractions and of implicit fractions, like $\tan(x) = \frac{\sin(x)}{\cos(x)}$.
3. Determine the intersection of all the parts of $\mathbb{R}$ obtained in the previous points.

The **range** of a function f consists of all values $f(x)$ it assumes when x ranges over its domain.

Domains and ranges are best described as unions of non-intersecting intervals and rays, e.g., $(-\infty, -4) \cup (-4, -1] \cup [0, 3]$.

Given two functions f and g, their **composition** $f \circ g$ is defined by
$$(f \circ g)(x) = f(g(x)),$$
so x must be in the domain of g and $g(x)$ in the domain of f for this to make sense.

Questions

1. Consider the functions $f(x) = \frac{x^2 + 2x}{x}$ and $g(x) = \sqrt{(x+1)^2} + 1$. Show that these functions are different. Find a ray on which they are equal.
2. Consider the functions $f(x) = 2\sqrt{x-1} + \frac{1}{x-1}$ and $g(x) = -\frac{1}{x-1}$. Write the following functions in as simple a form as possible, and determine their domain:
   (a) $f + g$;
   (b) $\frac{g}{f}$.
3. Find the range of the function $f(x) = \frac{2x^2 + 1}{x^2 + 1}$.
4. Consider the functions $f(x) = \sqrt{x} + x + 1$ and $g(x) = x + \frac{1}{x}$. Determine their compositions $f \circ g$ and $g \circ f$, and their domains.
5. Consider the piecewise defined function
$$u(t) = \begin{cases} t^2, & t \geq 0, \\ 1, & -2 < t < 0, \\ -\frac{1}{2}t, & t < -2. \end{cases}$$
   What are its domain and its range? Plot its graph. Also, plot the graph of the function $v(t) = u(t+1) - 2$.