Homework/Tutorial 1

Please hand in your work at the end of the tutorial. Make sure you put your name and student ID number on what you hand in.

A complete solution to every question is worth 2 marks.

What this homework is about

Module MA1S11A is devoted to functions. In the exercises below you are invited to recall the basic functions you already know—polynomials and trigonometric functions. The argument x is considered to be a real number in all the exercises.

An important part of analysing a function is to determine when it takes positive/zero/negative values. This is what you will do below.

Reminder

The roots of the polynomial $P(x) = ax^2 + bx + c$, where $a \neq 0$, are computed by the formula

$$x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The polynomial then factorises as

$$P(x) = a(x - x_{-})(x - x_{+}).$$

If x_1 is a root of a polynomial Q(x) of degree n, then Q decomposes as $Q(x) = (x - x_1)R(x),$

where R(x) is a polynomial of degree n-1.

Questions

- 1. Find the roots of the polynomial $P(x) = x^2 5x + 4$, and determine all the values of the arguments x for which P(x) > 0.
- 2. Check that x = 1 is a root of the polynomial $Q(x) = x^3 6x^2 + 9x 4$. Divide Q(x) by x 1. Use this to factorise Q(x), and to determine all x for which $Q(x) \ge 0$.
- 3. Find all x for which $\tan(x + \frac{\pi}{4}) < 0$.
- 4. Find all x for which $-3\sin(x)\sin(2x) \ge 0$.
- 5. Describe all x for which
 - (a) $-7 \le x < 2$ and $-1 < x \le 12;$
 - (b) $-7 \le x < 2$ or $-1 < x \le 12$.