UNIVERSITY OF DUBLIN
TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

MA1S11A (CALCULUS): SAMPLE EXAM

ATTEMPT **FOUR** QUESTIONS

For each task, the number of points you can get for a complete solution of that task is printed next to it. (A complete solution includes coherent explanations for answers you give.)

Unless otherwise specified, you may use all statements proved in lectures without proof; when using some statement, you should formulate it clearly.

⚠ This sample exam is given to you to familiarise yourselves with the structure of the exam document. The questions in the real exam will be much closer to your homework and to the examples seen in class. The topics the most likely to be covered are

- analysis (symmetries, extremal and inflection points etc.) and graphing of a function (following the algorithm you were given for rational functions, cf. Homework 8);
- equations of tangents to the graph of a function (Homework 6);
- area between two graphs (Homework 10);
- arc length of a curve (Lecture 27);
- minima and maxima of a function (Homework 9, Lecture 17);
- inverse functions: proving their existence, and computing their derivatives (Homeworks 3, 4, 7).

⚠ This list is not exhaustive! The best way to prepare yourself for the exam is to go through all the examples considered in class and through the solutions to all the homeworks.

You are supposed to know and understand all the basic formulas and theorems. Computations will be easy, so you will not be allowed to use calculators.

1. (a) (10 points) Determine which of the following three curves in the x-y plane are symmetric about the x-axis, symmetric about the y-axis, symmetric about the origin: $y = x^3 + \tan x$, $y^3 + e^{x^2}\cos\frac{1}{x} = 0$, $y^2 + 5xy - x^2 = 0$. For each of these curves also indicate whether it is a graph of some function of x.

   (b) (5 points) Compute the limit $\lim\limits_{x\to 0} \dfrac{\sin(7x)}{e^{2x+1} - e}$.

   (c) (5 points) Given a real number t, compute the limit $\lim\limits_{x\to+\infty} \frac{3(1-t)(tx)^4}{x^4+1}$. (The answer might depend on t.)

   (d) (5 points) For what t does the limit from the previous point take its maximal value?

2. Let us consider the function $f(x) = x^3 - 10x^2 + 25x$.

   (a) (7 points) Determine relative extrema and the intervals of monotonicity of $f(x)$.

   (b) (7 points) Determine inflection points and the intervals of concavity up/down of the graph of $f(x)$.

   (c) (7 points) Describe the behaviour of f at $\pm\infty$.

   (d) (4 points) Based on results obtained in the tasks above, plot a rough sketch of the graph of $f(x)$.

3. (a) (10 points) Show that the function $f(x) = \sin(x^2)$ has an inverse on $[0, \sqrt{\frac{\pi}{2}}]$. Compute the derivative of this inverse f^{-1} where it exists.

   (b) (10 points) Write down the equation of the tangent to the graph of the function f above at the point $x = \sqrt{\frac{\pi}{6}}$. Using this, write down the equation of the tangent to the graph of f^{-1} at the point $x = \frac{1}{2}$.

   (c) (5 points) Is the piecewise defined function

$$f(x) = \begin{cases} \frac{x^2-7x+6}{x-1}, x \neq 1, \\ -6, x = 1, \end{cases}$$

   continuous at $x = 1$? If not, what is its discontinuity type? Explain your answers.

4. Evaluate the integrals

   (a) (5 points) $\int \frac{\sin 2\theta}{1-\cos 2\theta}\, d\theta$;

   (b) (5 points) $\int_{-1}^{1} \frac{x\,dx}{\sqrt{1+x^2}}$;

   (c) (7 points) $\int_{1}^{2} x \ln x\, dx$;

   (d) (8 points) $\int x^2 \sqrt[3]{1+3x}\, dx$.

5. (a) (12 points) Determine the equations of tangent lines to the graphs $y = xe^x$ and $y = (x^2 - x)e^x$ at the point $(0,0)$ belonging to both of these graphs.

   (b) (13 points) Determine the area between the graphs $y = xe^x$ and $y = (x^2 - x)e^x$.