MAU22302/33302 - Euclidean and non-Euclidean Geometry

Tutorial Sheet 9

Trinity College Dublin

Course homepage

Exercise 1 *Cross ratio and infinity*

i) Compute the following cross ratios

$$[1,2,3,4],\ [2,5,8,20],\ [4,\infty,0,6]$$

ii) Compute $[x_2,x_1,x_4,x_3]$ and $[x_3,x_4,x_1,x_2]$ in terms of $[x_1,x_2,x_3,x_4]$.

iii)

iv) Determine expressions for

$$[\infty,x_2,x_3,x_4],\ [x_1,\infty,x_3,x_4],\ [x_1,x_2;\infty,x_4],\ [x_1,x_2,x_3,\infty]$$

in terms of the finite coordinates

v) Determine a formula for the cross ratio in terms of homogeneous coordinates

$$\big[[x_1:y_1],[x_2:y_2],[x_3:y_3],[x_4:y_4]\big] = ?$$

Start with the case of finite coordinates, and check it works for one with $y_i = 0$ *for some* i.

Exercise 2 *Constructing projective transformations*

Using the cross ratio, determine matrix associated to the following projective transformations $\mathbb{RP}^1 \to \mathbb{RP}^1$, given by specifying the images of three points:

i)
$$[1 : 1] \mapsto [2 : 1],\ [3 : 1] \mapsto [5 : 1],\ [5 : 1] \mapsto [8 : 1]$$

ii)
$$[4 : 2] \mapsto [6 : 3],\ [15 : 3] \mapsto [8 : 2],\ [100 : 25] \mapsto [255 : 15]$$

iii)
$$[1 : 0] \mapsto [0 : 1],\ [3 : 6] \mapsto [1 : 1],\ [8 : 5] \mapsto [3 : 1]$$

Exercise 3 *Bonus: the complex projective line*

Entirely analogously to the real projective line, we can define the complex projective line $\mathbb{CP}^1$ as the set of all (complex) lines in $\mathbb{C}^2$ through the origin, and identically define homogeneous coordinates, and points at infinity. We can also establish a bijection with a more familiar space. Determine this familiar space.

Hint: How many points at infinity are there? What does the rest of the space resemble? Where have we here the word projection before?