MAU22302/33302 - Euclidean and non-Euclidean Geometry

Tutorial Sheet 5

Trinity College Dublin

Course homepage

Exercise 1 *The National Gallery is inefficient*

In lectures, we showed that a polygonal room with n vertices could be effectively guarded by a group of $\lfloor \frac{n}{3} \rfloor$ guards. By explicitly constructing an example for every $n = 3m$, show that this is the minimal number of guards we can hire without more knowledge of the floor plan than the number of corners.

Hint: The National Gallery has a room that's not too far off a worst case scenario. If that doesn't help, try a few small examples with have points that can only be seen from small areas, and glue those together so that guards views don't overlap too much

Exercise 2 *Small lattice polygons*

1. Given a lattice polygon P (on $\mathbb{Z}^2$) of area 4, what are the possible values of (I_P, B_P)?

2. Does there exist a lattice polygon of area 4 with each of these interior/boundary vertex counts?

Exercise 3 *Filling in the gaps*

We proved a generalisation of Pick's theorem to lattice polygons with holes by introducing the Euler characteristic and using this to count elementary triangles.

$$\text{Area}(P) = I_P + \frac{B_P}{2} - \chi(P)$$

where

$$\chi(P) = |V(P)| - |E(P)| + (1 - h(P))$$

A more direct proof would be via subtraction

1. By inducting on the number of holes, and using Pick's theorem for polygons, derive a formula for the area of a lattice polygon with holes in terms of the number of interior points, boundary points, and holes.

2. Hence, give an alternative explanation as to why $\chi_{\mathcal{T}}(P)$ depends only on P, at least for elementary triangulations $\mathcal{T}$.