MAU22302/33302 - Euclidean and non-Euclidean Geometry

Tutorial Sheet 4

Trinity College Dublin

Course homepage

Exercise 1 *Reflection decompositions*

Let's decompose some examples of isometries of $\mathbb{R}^2$ into reflections.

1. Consider the isometry $x \mapsto x + (1, 1)$. Can you write this as a composition of two reflections? Can you find more than one pair of reflections whose composition is this translation?

2. Consider the rotation by $\pi/3$ anticlockwise around $(0, 0)$. Can you write this as a composition of two reflections? Can you find more than one pair of reflections whose composition is this rotation?

3. Can you write the rotation by $\pi/3$ anticlockwise around $(1, 1)$ as a composition of reflections? Can you do it with only two reflections?

Exercise 2 *Triangulations of polygons*

1. How many distinct strong triangulations are there of a regular n-gon for $n = 3, 4, 5, 6$? What if we only consider strong triangulations distinct up to isometries (i.e. rotation and reflection)?

2. Show that for any polygon P with n vertices, the sum of the internal angles is $(n - 2)\pi$.

Exercise 3 *If you have time for further reflection...*

We define reflection in $\mathbb{R}^3$ as follows. Suppose we have a plane P and a point x. If $x \in P$, then the reflection of x in P is x. Otherwise consider the normal line to P through x, intersection P at y. The reflection in P of x is then the unique other point $\tilde{x}$ on the normal line such that $d(x, y) = d(\tilde{x}, y)$. How could we show that reflection is an isometry? Can we reduce to the case of $\mathbb{R}^2$? Would this piggybacking let us show that reflection is an isometry in $\mathbb{R}^n$ for all $n \geq 2$?