MAU34106 - Galois Theory

Exercise Sheet 4

Trinity College Dublin

Course homepage

Answers are due for Friday, April 11th, 17:00.
The use of electronic calculators and computer algebra software is allowed.

Exercise 1 *Please don't compute the discriminants by hand (100pt)*

Determine the Galois groups of the splitting fields of the following polynomials. You may use without proof the following results. Note that not every polynomial is necessarily irreducible.

- For a cubic polynomial $f(x) = x^3 + ax^2 + bx + c$, the discriminant is given by
$$a^2b^2 - 4b^3 - 4a^3c - 27c^2 + 18abc.$$
- For a quartic polynomial $f(x) = x^4+ax^3+bx^2+cx+d$, the discriminant is given by
$$\begin{aligned} &256d^3 - 192acd^2 - 128b^2d^2 + 144bc^2d - 27c^4 + 144a^2bd^2 \\ &\quad - 6a^2c^2d - 80ab^2cd + 18abc^3 + 16b^4d - 4b^3c^2 \\ &\quad - 27a^4d^2 + 18a^3bcd - 4a^3c^3 - 4a^2b^3d + a^2b^2c^2 \end{aligned}$$

You are also welcome to, and encouraged to, use a computer to determine discriminants.

1. (25pts) $x^3 - 4x + 6$
2. (25pts) $x^3 - x^2 + x - 1$
3. (25pts) $x^4 - 2x^3 + 2x^2 + 2$
4. (25pts) $x^4 - 4x + 2$

Hint: You shouldn't need to think about primes greater than 5.

This exercise is entirely optional. If submitted before the deadline, your continuous assessment will consist of the best 3/4 assignments submitted. No extensions will be given

Further exercises on this topic can be found on the course webpage, and, I strongly encourage you to give them a try, as the best way to learn maths is through practice.

They are arranged by theme, and roughly in order of difficulty within each theme, with the first few working as good warm-ups, and the remainder being of similar difficulty to the main exercise. You are welcome to email me if you have any questions about them. The solutions will be made available alongside the problems