2D foams above the jamming transition: Deformation matters

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Simulations of 2D foams above the jamming point suggest a different form for the variation of the average contact number with packing fraction from that in soft disk systems.

The soft disk simulation

- Soft disk interaction dependent on overlap δ_{ij}:

$$U_{\text{soft}} = \frac{1}{2}\sum_{i<j}^{N} \delta_{ij}^2$$

- Start from random initial configuration

Energy minimisation: Conjugate Gradient

- Find energy minimum for given packing fraction ϕ
- Disks do not deform upon contact

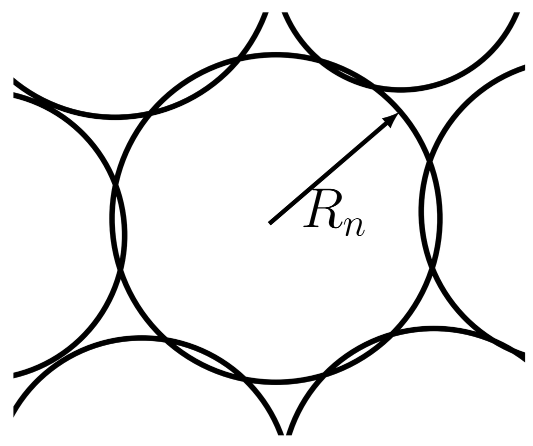

Fig. 1: Sample disk with overlaps.

Packing Fraction ϕ:

- $\phi = {}^1/_{L^2} \sum_n \pi R_n^2$

The PLAT simulation for 2D foam

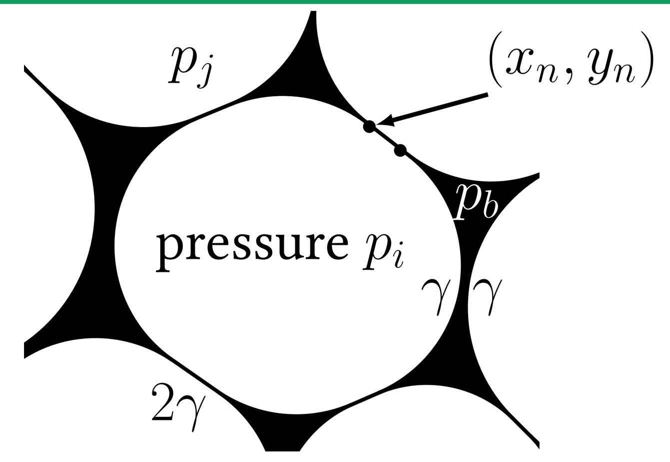

Fig. 2: Foam simulation in PLAT [1, 2].

Packing Fraction ϕ:

- Gas fraction

Poly dispersity:

- $\langle a^2 \rangle / \langle a \rangle^2 = 1.02$

Laplace–Young law:

- For liquid–gas interfaces $\Delta p_{ib} = \gamma / r_{ib}$
- For bubble films $\Delta p_{ij} = 2\gamma / r_{ij}$
- Lines are circular arcs with radius r_{ij}

Variables:

- Vertex coordinates (x_n, y_n)
- Cell and Plateau border pressures

Constraints for equilibrium:

- Given cell area
- Arcs meet tangentially at a vertex

Deformation:

- Bubbles change shape upon contact

Compression of soft disks

wet ⟵⟶ dry

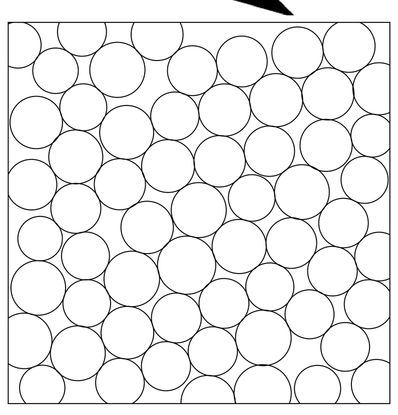

Fig. 3: $\phi = 0.84$

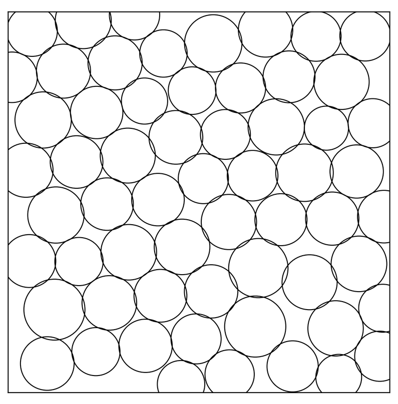

Fig. 4: $\phi = 0.90$

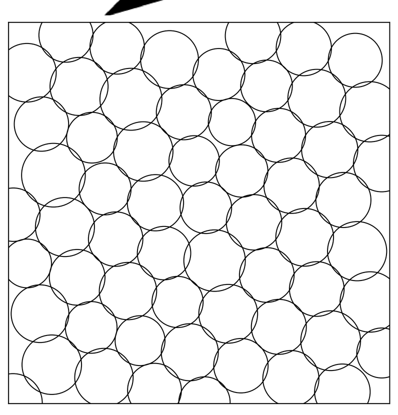

Fig. 5: $\phi = 1.00$

Compression of foam

wet ⟵⟶ dry

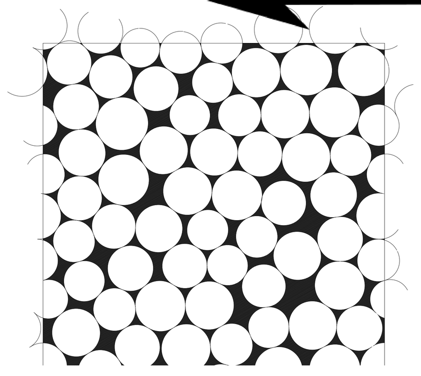

Fig. 6: $\phi = 0.84$

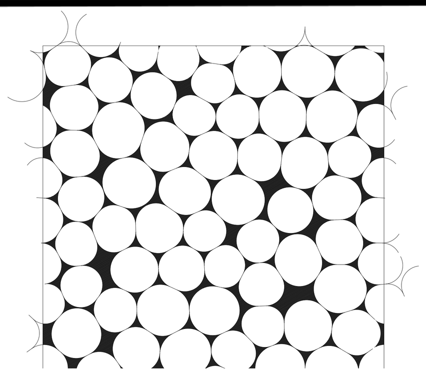

Fig. 7: $\phi = 0.89$

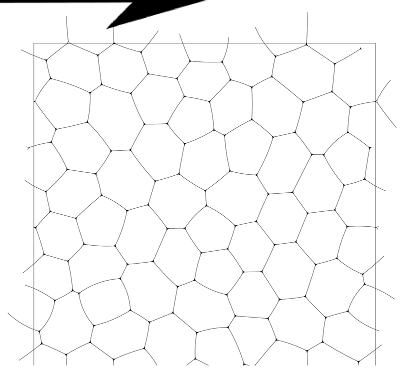

Fig. 8: $\phi = 0.99$

Soft disks: Square root increase in $Z(\phi)$

$Z(\phi)$ = **Average contact number**

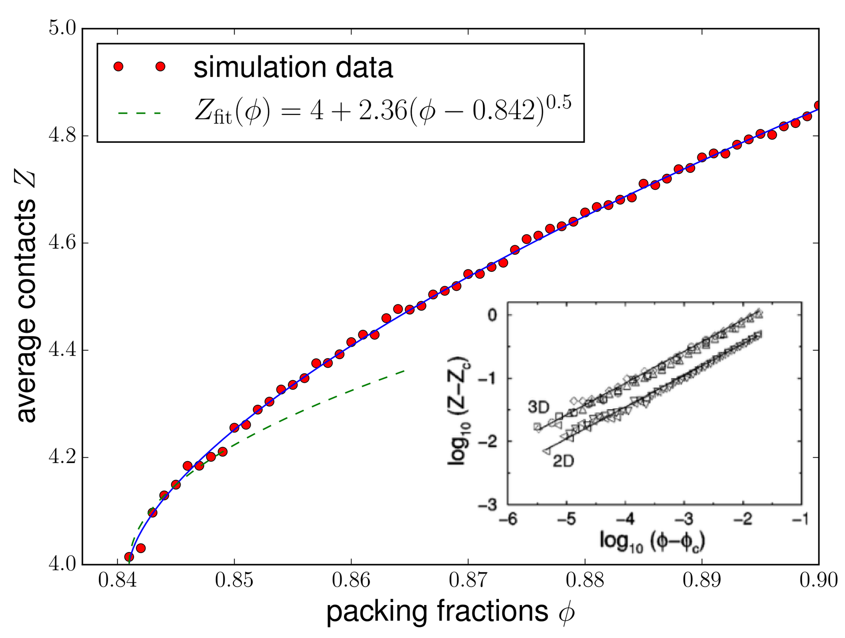

Fig. 9: $Z(\phi)$ in soft disk systems (inset plot from [6]).

- $Z - Z_c \propto \sqrt{\phi - \phi_c}$
- Well known behaviour in soft disk systems [8, 6, 10]
- Square-root scaling also observed in experimental data for photo-elastic disks [4]
- Power law relationship $Z(\phi)$ for a confined bubble raft experiment [3] (Identification of contacting bubbles? Definition of liquid fraction?)

PLAT: Linear increase in $Z(\phi)$

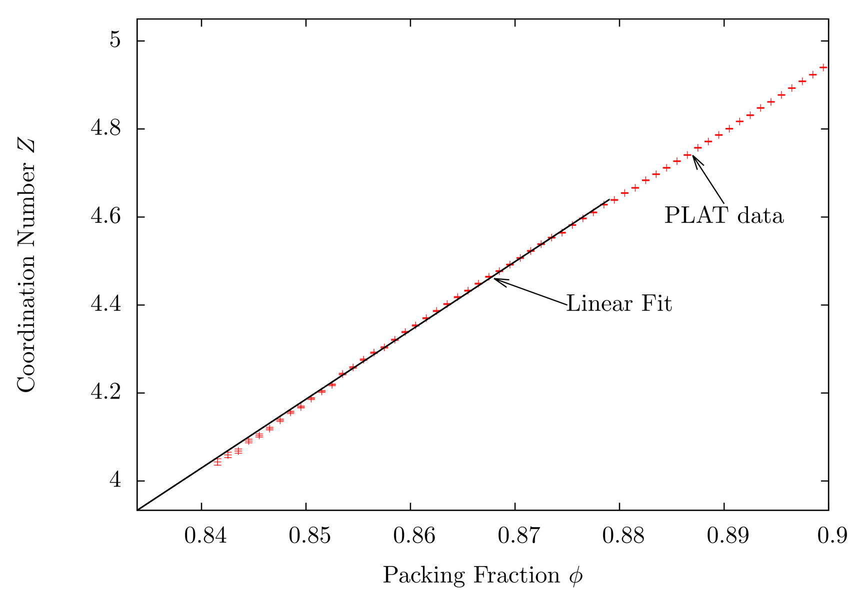

Fig. 10: $Z(\phi)$ in PLAT simulation.

- $Z - Z_c \propto \phi - \phi_c$
- Periodic system: $Z_c = 4(1 - {}^1/_N)$
- data averaged over 10,000 simulations of $N = 60$ bubble systems
- hints of this in earlier simulations (PLAT [1] and lattice gas model [9])

The separation between bubbles at the critical packing fraction

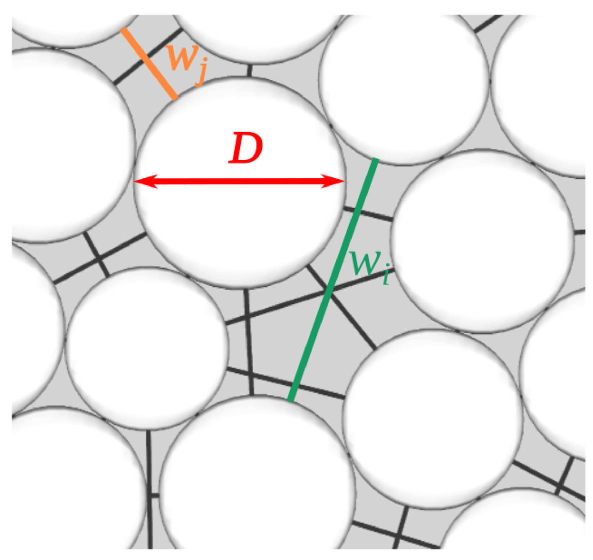

Fig. 11: Separations w_i between different disks/bubbles.

Is $Z(\phi)$ determined by the distribution separation between bubbles?

Crude argument [7]: For affine compressions at ϕ_c, $Z(\phi) - Z_c$ is given by the radial integral over the distribution of separation $f(w)$ in the limit $\epsilon \approx {}^w/_D \to 0$

$$Z(\phi) - Z_c = 2\pi \int_0^{\epsilon D} \mathrm{d}w\, f(w)(w + D)\,, \qquad \text{compression: } \epsilon = \frac{\phi - \phi_c}{2\phi_c}$$

ϕ_c: critical packing fraction. D: average diameter.

Soft disks:

$$f(w) \propto \left(\frac{w}{D}\right)^{-\frac{1}{2}} \Rightarrow Z - Z_c \propto \sqrt{\phi - \phi_c}$$

2D foams:

$$f(w) = \text{const.} \Rightarrow Z - Z_c \propto \phi - \phi_c$$

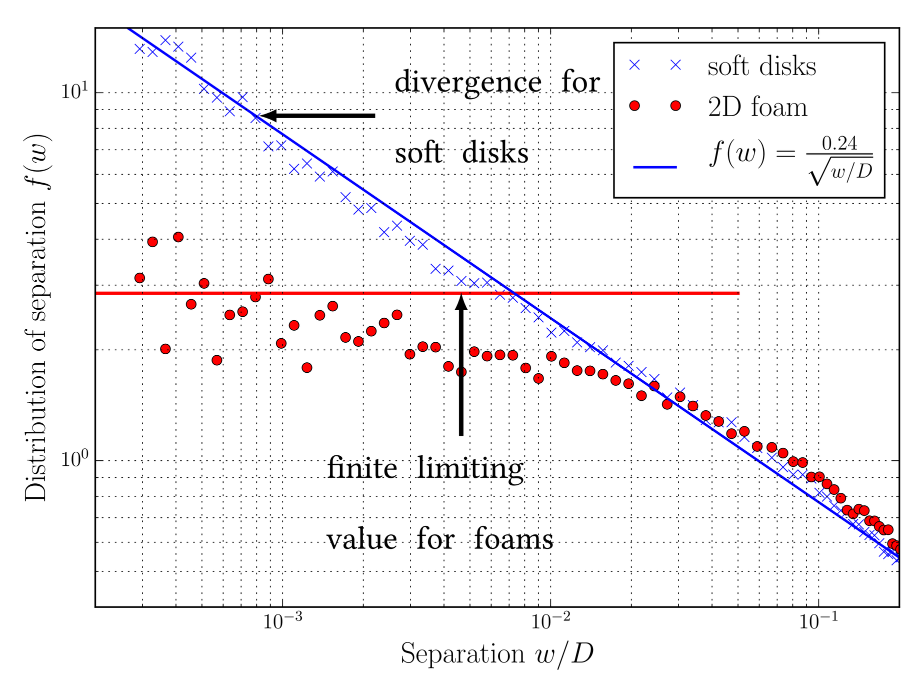

Fig. 12: Distribution of separation.

References

[1] F. Bolton and D. Weaire. *The effects of Plateau borders in the two-dimensional soap froth I. Decoration lemma and diffusion theorem.* In: *Phil. Mag. B* 63 (1991), pp. 795–809. [2] F. Bolton and D. Weaire. *The effects of Plateau borders in the two-dimensional soap froth. II. General simulation and analysis of rigidity loss transition.* In: *Phil. Mag. B* 65 (1992), pp. 473–487. [3] G. Katgert and M. van Hecke. *Jamming and geometry of two-dimensional foams.* In: *EPL Europhys. Lett.* 92 (2010), p. 34002. [4] T. S. Majmudar et al. *Jamming Transition in Granular Systems.* In: *Phys. Rev. Let.* 98.5 (Jan. 2007). [5] C. B. O'Donovan, E. I. Corwin, and M. E. Möbius. *Mean-field granocentric approach in 2D & 3D polydisperse, frictionless packings.* In: *Philos. Mag.* 93 (2013), pp. 4030–4056. [6] C. S. O'Hern et al. *Jamming at zero temperature and zero applied stress: The epitome of disorder.* In: *Phys. Rev. E* 68 (2003). [7] A. O. Siemens and M. van Hecke. *Jamming: A simple introduction.* In: *Physica A: Statistical Mechanics and its Applications* 389.20 (2010), pp. 4255–4264. [8] L. E. Silbert et al. *Geometry of frictionless and frictional sphere packings.* In: *Phys. Rev. E* 65 (2002). [9] Q. Sun and S. Hutzler. *Lattice gas simulations of two-dimensional liquid foams.* In: *Rheo. Acta* 43.5 (2004), pp. 567–574. [10] M. van Hecke. *Jamming of soft particles: geometry, mechanics, scaling and isostaticity.* In: *J. Phys. Condens. Matter* 22 (2010), p. 033101.

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