2D foams above the jamming transition: Deformation matters

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Simulations of 2D foams above the jamming point suggest a different form for the variation of the average contact number with packing fraction from that in soft disk systems.

The soft disk simulation

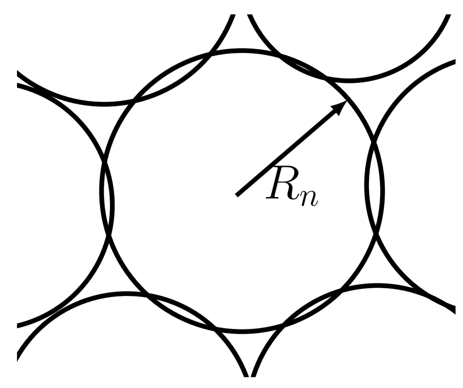


Fig. 1: A soft disk with corresponding overlaps.

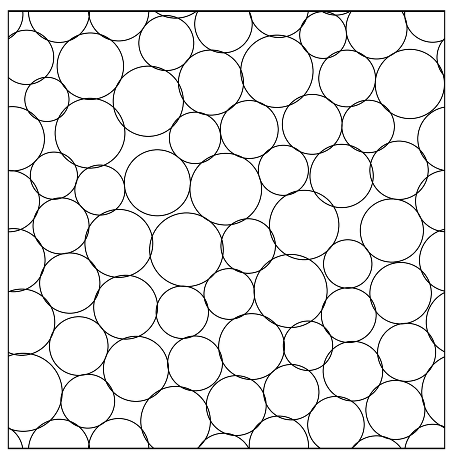


Fig. 2: Soft disk packing at $\phi = 0.90$.

Interaction and minimisation:

- Random initial configuration at packing fraction ϕ
- Interaction dependents on overlap δ_{ij}:

$$U_{\text{soft}} = \frac{1}{2}\sum_{i<j}^{N} \delta_{ij}^2$$

- **Energy minimisation: Conjugate Gradient**

Deformation:

- Disks do not deform
- No cell area conservation

The `Plat` simulation for 2D foam

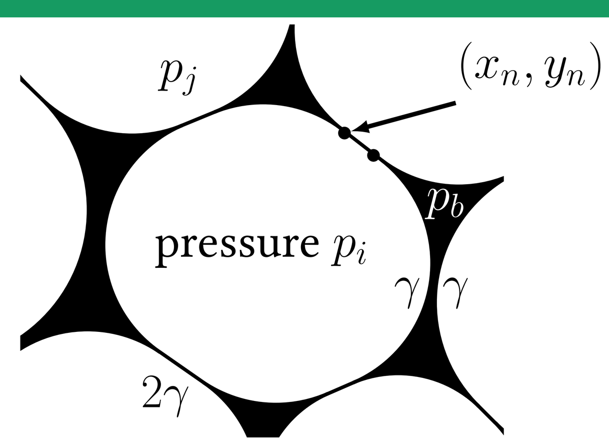


Fig. 3: A deformed bubble in `Plat` [1, 2].

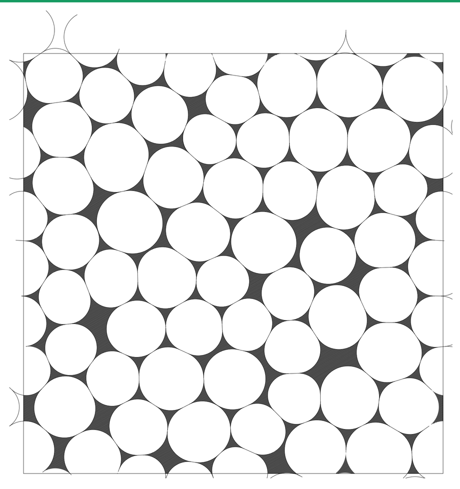


Fig. 4: `Plat` foam at $\phi = 0.90$.

Variables:

- Vertex coordinates (x_n, y_n)
- Cell and Plateau border pressures

Constraints for equilibrium:

- Const. cell area
- Circular arcs meet tangentially at a vertex (x_n, y_n)
- *Laplace–Young law* is fulfilled for cells and Plateau borders

Deformation:

- Bubbles change shape upon contact

The Morse–Witten theory for 2D bubbles

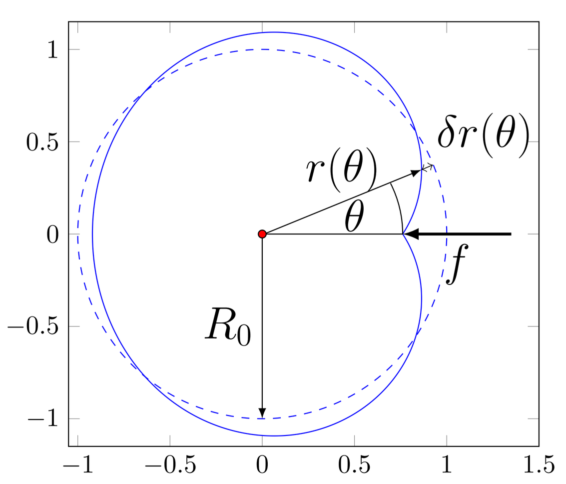


Fig. 5: Deformation of a 2D bubble with point force.

$$r(\theta) = R_0(1 + \delta r(\theta))$$

What is the shape $\delta r(\theta)$ of a single, incompressible 2D bubble, subject to a single contact force f?

Linear differential equation for the $\delta r(\theta)$:

$$-\left(\frac{\mathrm{d}^2}{\mathrm{d}\theta^2}\right)\delta r(\theta) = A + \frac{f}{\pi}\cos\theta$$

Solution for $\delta r(\theta)$:

$$\delta r(\theta) = \frac{f}{2\pi} g(\theta)\,, \quad g(\theta) = (\pi - \theta)\sin\theta - \frac{\cos\theta}{2} - 1$$

- δr linear in $f \Rightarrow$ superposition for multiple f

D. Weaire, R. Höhler, and S. Hutzler. *Bubble-bubble interactions in a 2d foam, close to the wet limit.* Adv. Colloid Interface Sci, accepted, (2017). `https://dx.doi.org/10.1016/j.cis.2017.07.004`.

The Morse–Witten (MW) simulation model

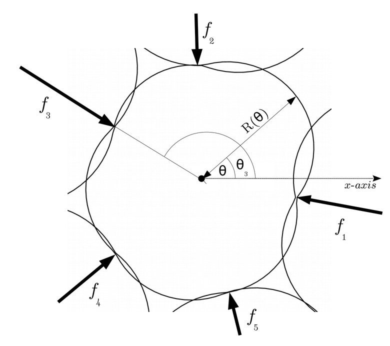


Fig. 6: Bubble deformation with multiple forces.

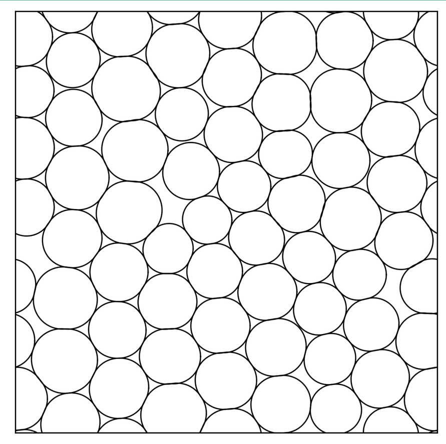


Fig. 7: MW foam at $\phi = 0.90$.

Shape for multiple forces:

$$\delta r(\theta) = \frac{1}{2\pi}\sum_c f_c g(\theta_c - \theta)$$

Equilibration procedure

1. Identify contacts, update δr
2. Calc. forces from δr, inverting Eq.
3. Balance forces at contacts
4. Move bubble centers in force direction
5. Recalculate δr with Eq.

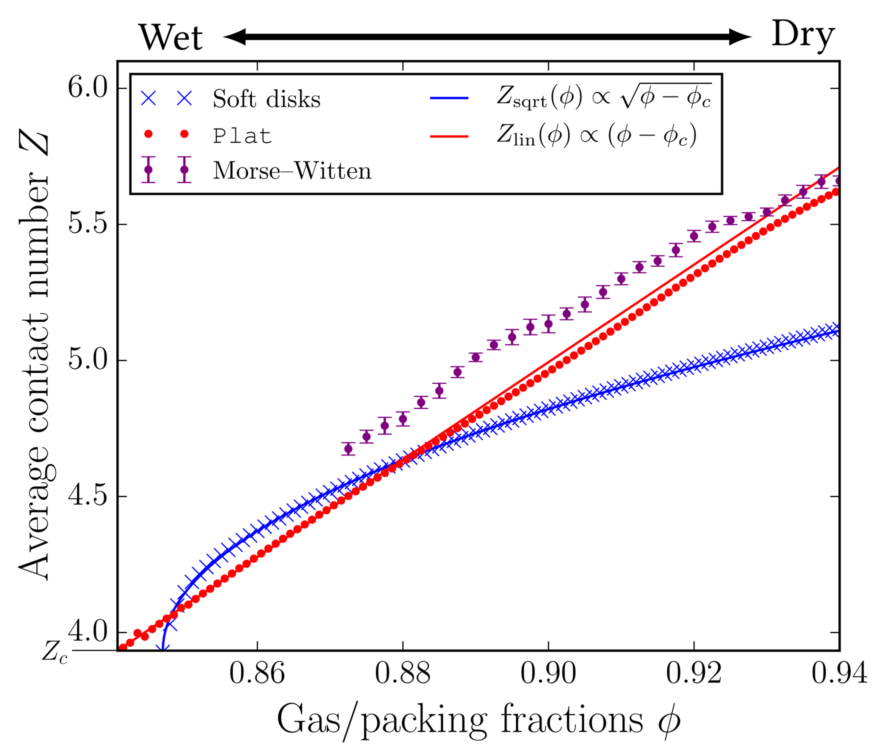
Constraints for equilibrium

- $\delta r(\theta)$ consistent with f_i
- force balance at films
- force balance on each bubble

The average contact number with increasing packing fraction $Z(\phi)$

Soft disk simulation:

- $Z - Z_c \propto \sqrt{\phi - \phi_c}$
- Well known behaviour in soft disk systems [11, 9, 13]
- Square-root scaling also observed in experimental data for photo-elastic disks [7]
- Power law relationship $Z(\phi)$ for a confined bubble raft experiment [6], *however*:
  - Identification of contacting bubbles?
  - Definition of liquid fraction?



`Plat` simulation:

- $Z - Z_c \propto \phi - \phi_c$
- Periodic system: $Z_c = 4(1 - 1/N)$
- data averaged over 10,000 simulations of $N = 60$ bubble systems
- hints of this in earlier simulations (`Plat` [1] and lattice gas model [12])

Morse–Witten (MW) model:

- Results are *preliminary*
- $Z - Z_c \approx \phi - \phi_c$
- data averaged over 5 simulations of $N = 100$ bubble systems

The separation between bubbles at the critical packing fraction

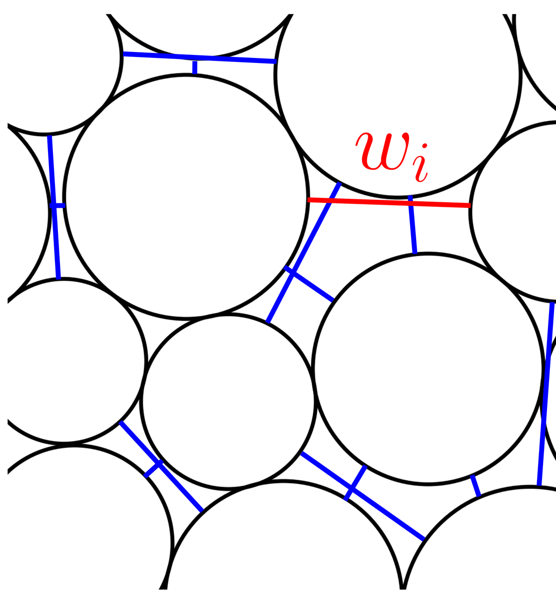


Fig. 8: Separations w_i between different disks/bubbles.

Is $Z(\phi)$ determined by the distribution separation between bubbles?

Crude argument [10]: For affine compressions at ϕ_c, $Z(\phi) - Z_c$ is given by the radial integral over the distribution of separation $f(w)$ in the limit $\epsilon \approx w/D \to 0$

$$Z(\phi) - Z_c = 2\pi \int_0^{\epsilon D} \mathrm{d}w\, f(w)(w + D)\,, \qquad \text{compression: } \epsilon = \frac{\phi - \phi_c}{2\phi_c}$$

ϕ_c: critical packing fraction. D: average diameter.

Soft disks:

$$f(w) \propto \left(\frac{w}{D}\right)^{-\frac{1}{2}} \Rightarrow Z - Z_c \propto \sqrt{\phi - \phi_c}$$

2D foams:

$$f(w) = \text{const.} \Rightarrow Z - Z_c \propto \phi - \phi_c$$

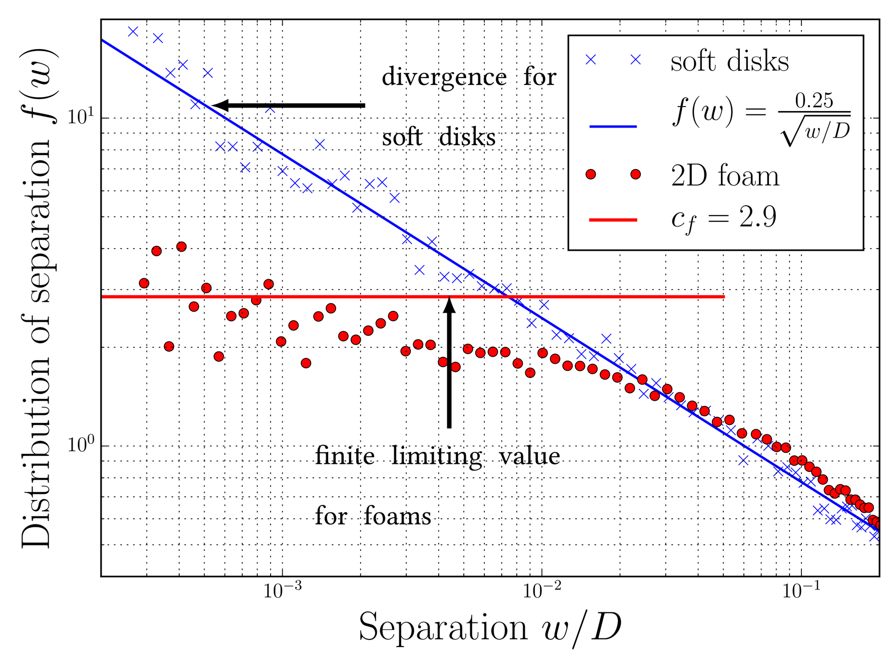


Fig. 9: Distribution of separation.

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