# Morse-Witten theory and its applications

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## Abstract taken from Morse and Witten (1993):

We discuss the deformation of a fluid droplet in an emulsion under external forces, such as those exerted by contact with neighbouring droplets. We find that the deformation energy associated with a small droplet-droplet contact scales as $f^2 \ln(1/f)$ with the force $f$ exerted between droplets. We consider the equation of state of an emulsion in which droplets are assumed to interact only via such contact forces, and obtain an osmotic compressibility which diverges logarithmically with the osmotic pressure in the limit of small pressure.

D. C. Morse and T. A. Witten. *Droplet elasticity in weakly compressed emulsions.* Europhys. Lett.,**22**, 549–555, (1993).

These important results have been poorly appreciated and hardly ever applied!

- Logarithmic form of interaction in 3D
- Extended system of bubbles or drop can be represented by a force network: see below

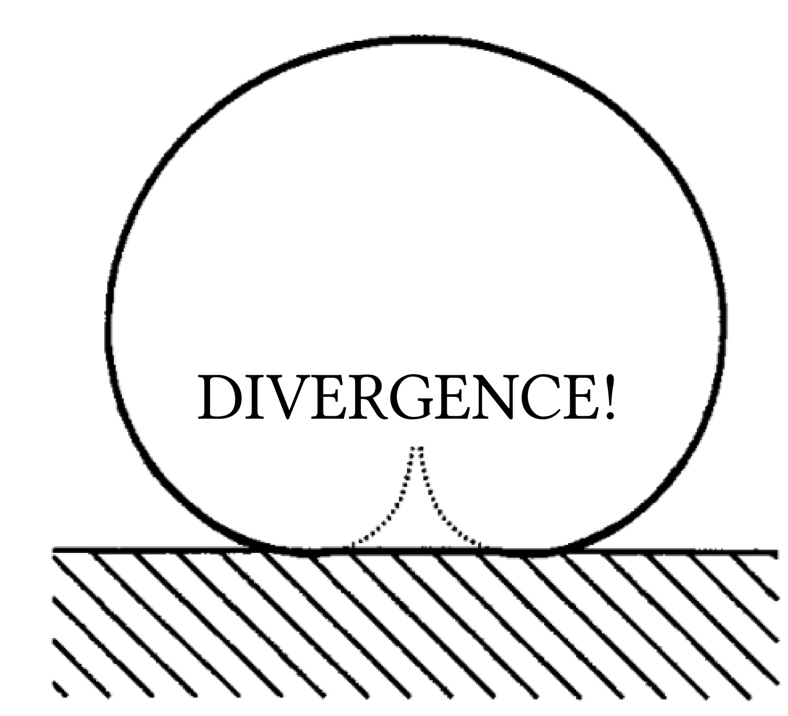



**Fig.:** Sessile drop.

## The Morse–Witten (MW) theory for 2D bubbles

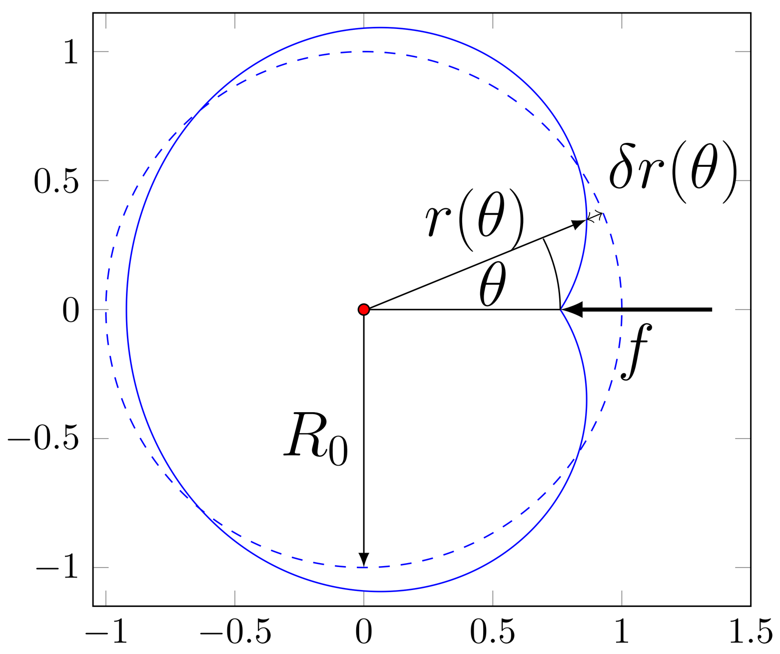



**Fig.:** Deformation of a 2D bubble with point force.

$$r(\theta) = R_0(1 + \delta r(\theta))$$

What is the shape $\delta r(\theta)$ of a single, incompressible 2D bubble, subject to a single contact force $f$ (and a compensating body force)?

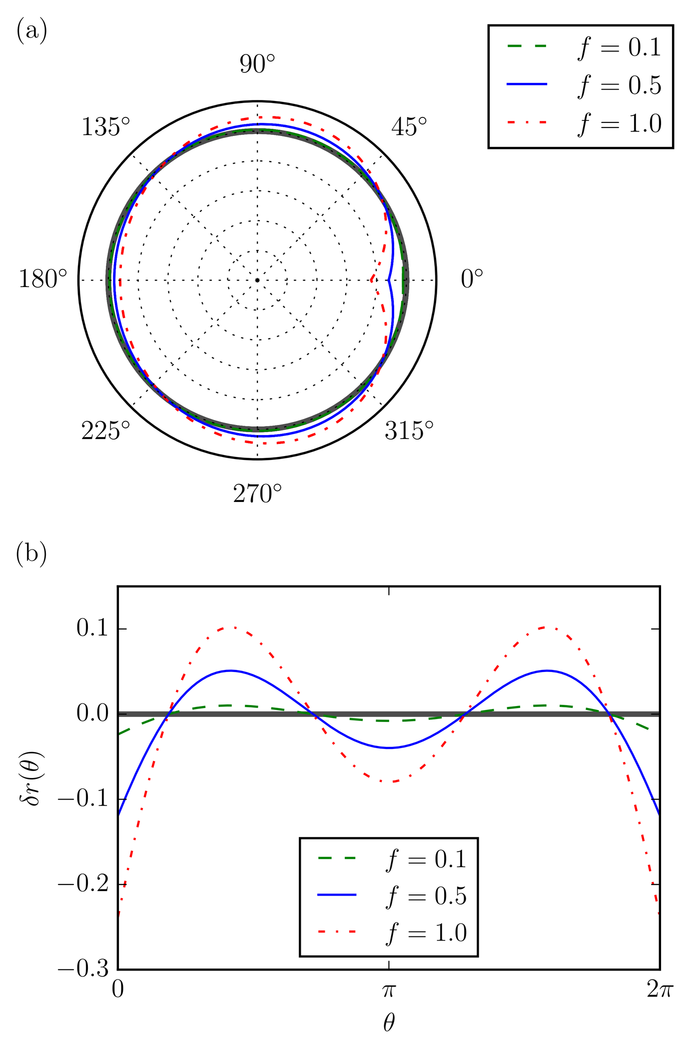
**Linear differential equation** for the $\delta r(\theta)$:

$$-\left(\frac{\mathrm{d}^2}{\mathrm{d}\theta^2}\right)\delta r(\theta) = A + \frac{f}{\pi}\cos\theta$$

**Solution** for $\delta r(\theta)$:

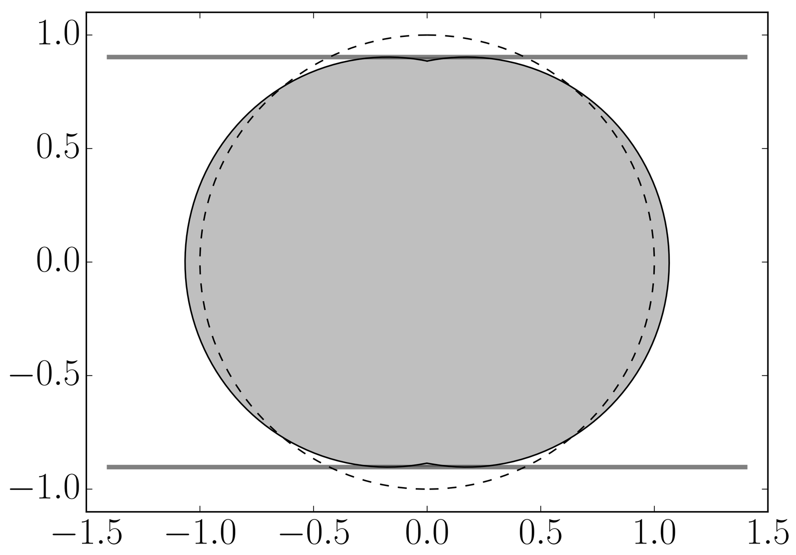
$$\delta r(\theta) = \frac{f}{2\pi} g(\theta)\,, \quad g(\theta) = (\pi - \theta)\sin\theta - \frac{\cos\theta}{2} - 1$$

- $\delta r$ linear in $f$
- Area $A = \pi R_0^2$ is conserved
- Change in area of the enclosed curve is of higher order $\mathcal{O}(f^3)$

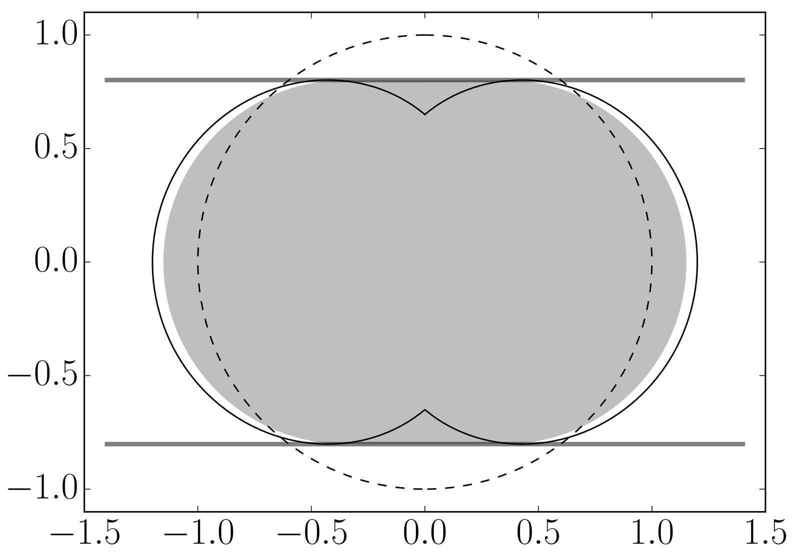



D. Weaire, R. Höhler, and S. Hutzler. *Bubble-bubble interactions in a 2d foam, close to the wet limit.*Adv. Colloid Interface Sci, accepted, (2017). `https://dx.doi.org/10.1016/j.cis.2017.07.004`.

## MW vs. exact: Bubble between plates

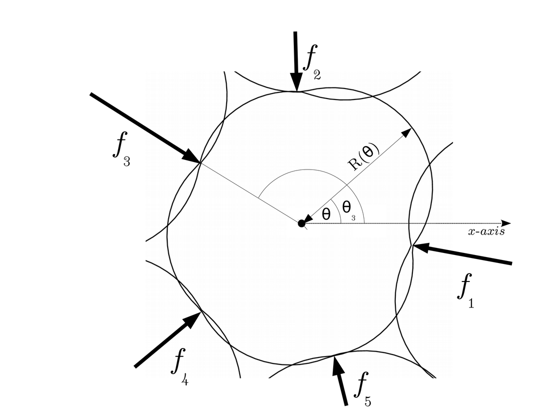


**(a)** Deformation of 10 %.

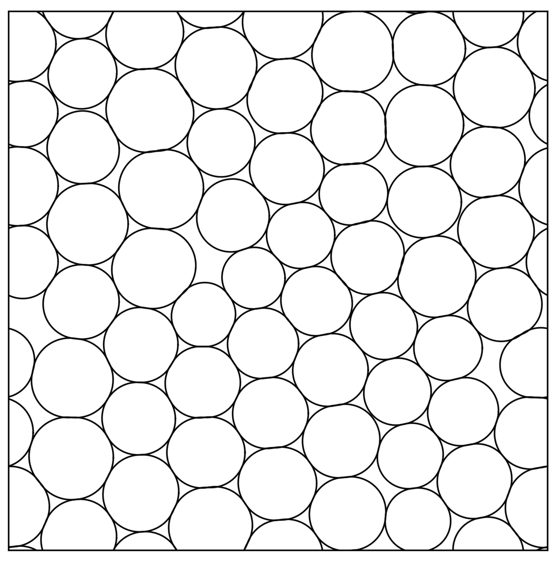


**(b)** Deformation of 20 %.

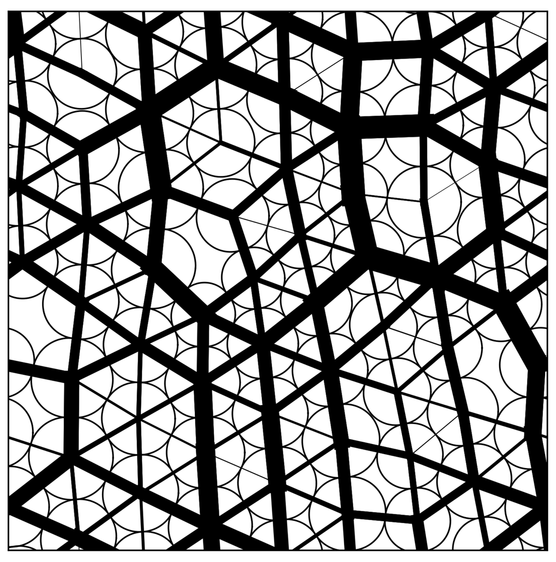
## MW applied to 2D disordered foam



**(a)** Bubble with multiple contact forces.

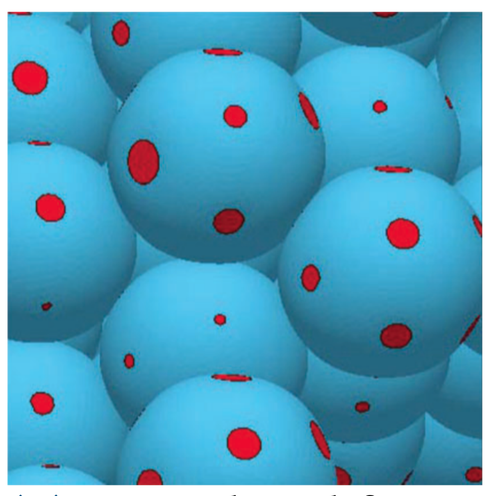


**(b)** 2D foam at $\phi = 0.9$.

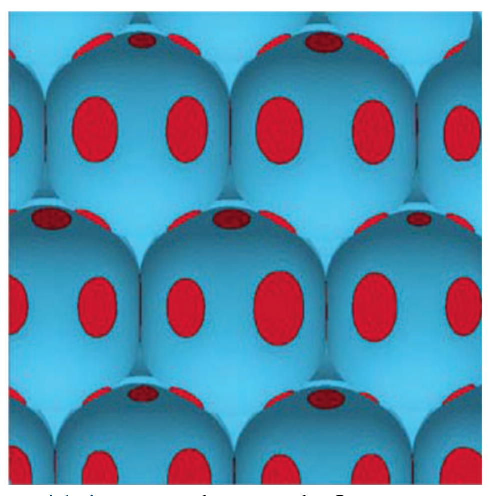


**(c)** Force network.

## MW applied to 3D foam



**(a)** Disordered foam.



**(b)** Ordered foam.

- Analysis of Surface Evolver simulations of foams reveals non-pairwise interaction between bubbles

R. Hoehler and S. Cohen-Addad. *Many-body interactions in soft jammed materials.* Soft Matter **13**, 1371-1383, (2017).

## The sessile and pendant drop

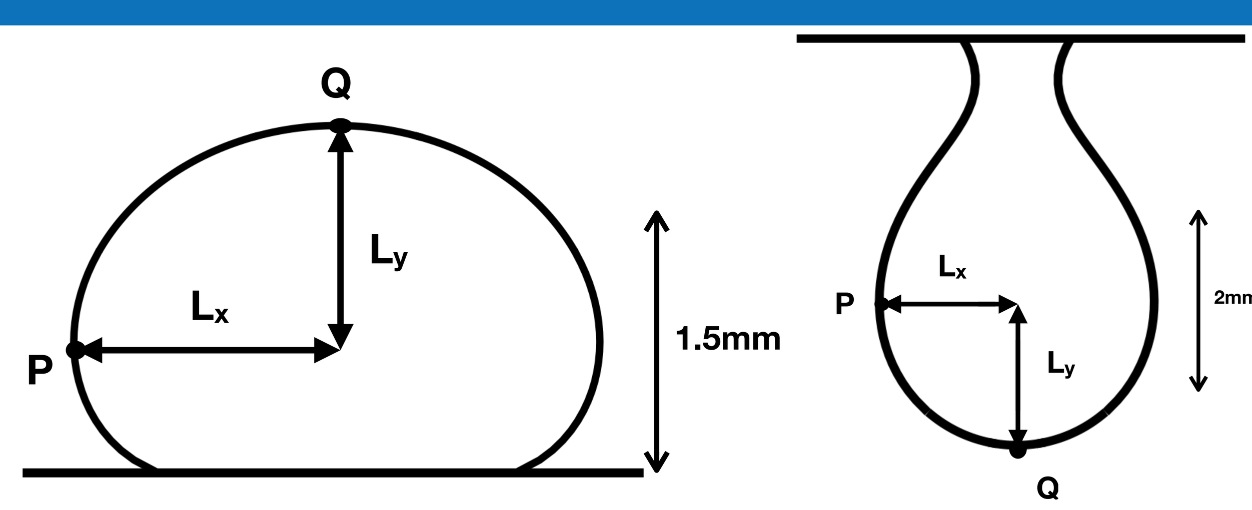



**Fig.:** The sessile drop (left) and pendant drop (right): proposed measurements

**Surface tension** $\sigma$:

$$\sigma = \frac{\ln 2}{24}\Delta\rho g \frac{(L_x + L_y)^3}{|L_x - L_y|}$$

**Simple** formula, needing only 2 length measurements.

S. Hutzler, J. Ryan-Purcell, D. Weaire, *A simple formula for the estimation of surface tension from two length measurements for a pendant or sessile drop*, in preparation, (2017).

A review on this subject is in preparation (R. Höhler, D. Weaire).

### References

[1] F. Bolton and D. Weaire. *The effects of Plateau borders in the two-dimensional soap froth I. Decoration lemma and diffusion theorem.* In: *Phil. Mag. B* 63 (1991), pp. 795–809. [2] F. Bolton and D. Weaire. *The effects of Plateau borders in the two-dimensional soap froth. II. General simulation and analysis of rigidity loss transition.* In: *Phil. Mag. B* 65 (1992), pp. 473–487. [3] F. F. Dunne et al. *Statistics and topological changes in 2D foam from the dry to the wet limit.* In: *Philosophical Magazine* 97.21 (2017), pp. 1768–1781. [4] R. Hoehler and S. Cohen-Addad. *Many-body interactions in soft jammed materials.* In: *Soft Matter* 13 (7 2017), pp. 1371–1383. [5] D. C. Morse and T. A. Witten. *Droplet Elasticity in Weakly Compressed Emulsions.* In: *EPL (Europhysics Letters)* 22 (1993), p. 549. [6] D. Weaire, R. Höhler, and S. Hutzler. *Bubble-bubble interactions in a 2D foam, close to the wet limit.* In: *Adv. Colloid Interface Sci* accepted (2017). [7] J. Winkelmann et al. *2D foams above the jamming transition: Deformation matters.* In: *Colloids Surf., A* (2017).

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