Quark–gluon vertex in a momentum subtraction scheme*

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We compute the quark-gluon vertex in quenched QCD, in the Landau gauge using an off-shell mean-field $\mathcal{O}(a)$-improved fermion action. The running coupling is calculated in an 'asymmetric' momentum subtraction scheme ($\widetilde{\text{MOM}}$). We obtain a crude estimate for $\Lambda_{\overline{\text{MS}}} = 170 \pm 65$ MeV, which is considerably lower than other determinations of this quantity. However, substantial systematic errors remain.

1. INTRODUCTION

A nonperturbative study of the quark–gluon vertex is of great interest for a number of reasons. Firstly, it allows us to determine the running coupling α_s from first principles, and also, by studying the large-momentum behaviour, to determine the scale parameter $\Lambda_{\overline{\text{MS}}}$. This approach is complementary to determinations of α_s from the three-gluon vertex [1], as well as numerous other methods [2–5].

Secondly, it may provide input for model studies of hadron structure, and in particular allow us to assess the reliability of truncation schemes for Dyson–Schwinger equations. The infrared behaviour may also yield information about dynamics of quark confinement [6].

Previously [7], the running coupling was studied in an asymmetric momentum subtraction scheme, and $\mathcal{O}(a)$ errors in the fermion action were found to be a serious problem. Here we expand on this study, using an off-shell $\mathcal{O}(a)$ improved quark propagator to reduce those errors.

In the continuum, the quark–gluon vertex with gluon momentum q and quark momenta $p, r = p+q$ can be decomposed as follows:

$$\Lambda_\mu(p,q) = \sum_{i=1}^{4} \lambda_i(p^2,q^2,r^2) L_{i,\mu}(p,q) + \sum_{i=1}^{8} \tau_i(p^2,q^2,r^2) T_{i,\mu}(p,q) \tag{1}$$

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The longitudinal components L_i and the transverse components T_i are given by [8]

$$L_{1,\mu} = \gamma_\mu \qquad L_{2,\mu} = \not{k} k_\mu \qquad L_{3,\mu} = k_\mu \qquad L_{4,\mu} = \sigma_{\mu\nu} k_\nu \tag{2}$$

$$\begin{aligned} T_{1,\mu} &= \ell_\mu & T_{2,\mu} &= \not{k}\ell_\mu \\ T_{3,\mu} &= q^2\gamma_\mu - \not{q} q_\mu & T_{4,\mu} &= \ell_\mu \sigma_{\nu\lambda} p_\nu q_\lambda \\ T_{5,\mu} &= \sigma_{\mu\nu} q_\nu & T_{6,\mu} &= -(qk)\gamma_\mu + \not{q} k_\mu \\ T_{7,\mu} &= -\frac{1}{2}(qk)\left[\not{k}\gamma_\mu - k_\mu\right] + k_\mu \sigma_{\nu\lambda} p_\nu q_\lambda \\ T_{8,\mu} &= -\gamma_\mu \sigma_{\nu\lambda} p_\nu q_\lambda - \not{p} q_\mu + \not{q} p_\mu \end{aligned} \tag{3}$$

where $k_\mu \equiv (2p+q)_\mu$, $\ell_\mu \equiv (pq)q_\mu - q^2 p_\mu$. We are particularly interested in λ_1, since this form factor is related to the running coupling. In the kinematical limit $q=0$, which we will be concentrating on here, all the transverse form factors τ_i, as well as λ_4, are zero. We will also be studying the 'symmetric' momentum configuration where $q=-2p$. In this case, all the form factors are zero apart from λ_1, τ_3 and τ_5.

2. RENORMALISATION

We impose 'continuum-like' MOM conditions on the quark and gluon propagators:

$$D^L(qa)\big|_{q^2=\mu^2} = \frac{Z_3(\mu,a)}{\mu^2} \tag{4}$$

$$S^L(pq)\big|_{p^2=\mu^2} = \left.\frac{Z_2(\mu,a)}{i\not{K}(p) + M(\mu)}\right|_{p^2=\mu^2} \tag{5}$$

where $K_\mu(p) \equiv \sin p_\mu$. We then impose momentum subtraction conditions on λ_1. We define the 'asymmetric' ($\widetilde{\text{MOM}}$) scheme by

$$\lambda_1^{\widetilde{\text{MOM}}}(\mu) \equiv \lambda_1(\mu^2,0,\mu^2) = \frac{1}{4}\operatorname{tr}\gamma_\nu\Lambda_\nu(p,0)\big|_{\substack{p^2=\mu^2 \\ p_\nu=0}} \tag{6}$$

where no sum over the Lorentz index ν is implied. It is also possible to define a 'symmetric' ($\overline{\mathrm{MOM}}$) scheme where $\lambda_1^{\overline{\mathrm{MOM}}}(\mu) \equiv \lambda_1(\mu^2, 4\mu^2, \mu^2)$; however, as we shall see it is not possible to implement this scheme in the Landau gauge on the lattice.

The MOM renormalised coupling is defined by

$$g_R(\mu) = iZ_2(\mu)Z_3^{1/2}(\mu)\lambda_1(\mu) \quad (7)$$

On the lattice, the proper vertex is given by

$$\begin{aligned} T_{\mu\nu}(q)\Lambda^a_\nu(p,q) &= T^a T_{\mu\nu}(q)\Lambda_\mu(p,q) \\ &\equiv \langle S(p)\rangle^{-1}\langle S(p)A^a_\mu(q)\rangle\langle S(p+q)\rangle^{-1}\langle D(q)\rangle^{-1} \end{aligned} \quad (8)$$

The tensor $T_{\mu\nu}$ is given by $D_{\mu\nu}(q) = T_{\mu\nu}(q)D(q)$. In Landau gauge, for $q \neq 0$ this is simply the transverse projector. Thus it is not possible to evaluate the longitudinal components of the quark–gluon vertex, including λ_1, for non-zero gluon momentum in Landau gauge. This means that our $\widetilde{\mathrm{MOM}}$ scheme is the only feasible scheme in this context.

3. RESULTS

We have analysed 495 configurations on a $16^3 \times 48$ lattice at $\beta = 6.0$, at one quark mass $ma = 0.058$, using the SW action with the mean-field $c_{\mathrm{sw}} = 1.479$. We have used the 'unimproved' quark propagator

$$S_0(x,0) \equiv (M^{-1})_{x0}; \quad S_0(p) \equiv \sum_x e^{-ipx}S(x,0) \quad (9)$$

and the 'improved' propagator [9]

$$S_I(p) = (1+b_q)S_0(p) + \lambda \quad (10)$$

with the mean-field coefficients $\lambda = 0.57, b_q = 1.14$. The configurations have been fixed to the Landau gauge with $\theta < 10^{-12}$.

In fig. 1 we show $\lambda_1^{\widetilde{\mathrm{MOM}}}(\mu)$ as a function of μ, for both S_0 and S_I. We see that there is a very big difference between the unimproved and improved quark propagators. However, this difference is almost entirely due to the tree-level behaviour of the improved propagator. It is possible to implement a tree-level correction scheme for the vertex similar to the one used for the quark propagator in [9]; however, that is not necessary in this case since the tree-level correction of the vertex is exactly cancelled by the tree-level correction of Z_2 given in [9].

Fig. 2 shows the running coupling $g_{\widetilde{\mathrm{MOM}}}(\mu)$ as a function of μ. We see that the results obtained from S_0 and S_I agree almost perfectly, despite the big difference in the unrenormalised λ_1 — confirming that the dominant (tree-level) behaviour is cancelled out at the renormalisation stage.

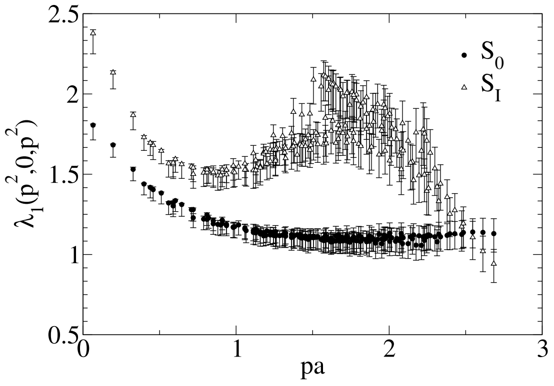

Figure 1. The unrenormalised $\lambda_1(p^2,0,p^2)$ as a function of pa. The filled circles are data obtained using the 'unimproved' propagator S_0, while the open triangles are obtained using the 'improved' propagator S_I.

We obtain $\Lambda_{\widetilde{\mathrm{MOM}}}$ by inverting the two-loop renormalisation group equation,

$$\Lambda = \mu e^{-\frac{1}{2b_0 g^2(\mu)}}\left(b_0 g^2(\mu)\right)^{-\frac{b1}{2b_0^2}} \quad (11)$$

The results from S_I are shown in fig. 3. It is not clear whether there is a perturbative window for this quantity. We do not expect two-loop perturbation theory to be valid until $\mu \gg 2$ GeV. It is therefore no surprise that we do not see a plateau in Λ until 3 GeV. More importantly, as observed in the three-gluon vertex [10], $\alpha_{\widetilde{\mathrm{MOM}}}$ contains power corrections, which we have not yet taken into account. We expect the inclusion of these corrections to significantly change the value of $\Lambda_{\widetilde{\mathrm{MOM}}}$. On the other hand, lattice artefacts become large for $\mu > 2$ GeV, so it is questionable whether we can trust our data here.

Z_2, Z_3 and $\lambda_1^{\widetilde{\mathrm{MOM}}}$ have been computed at one-loop level in the $\overline{\mathrm{MS}}$ scheme [11,8]. In Landau gauge, they are

$$Z_3(\mu) = 1 + \frac{\alpha_{\overline{\mathrm{MS}}}(\mu)}{4\pi}\frac{97}{72}C_A \qquad Z_2(\mu) = 1 \quad (12)$$

$$\lambda_1^{\widetilde{\mathrm{MOM}}}(\mu) = 1 + \frac{\alpha_{\overline{\mathrm{MS}}}(\mu)}{4\pi}\frac{C_A}{4}\left[3 + \frac{m^2}{\mu^2}\right] \quad (13)$$

From this we find, for $\mu \gg m$,

$$\alpha_{\widetilde{\mathrm{MOM}}}(\mu) = \alpha_{\overline{\mathrm{MS}}}(\mu)\left[1 + \frac{151}{12}\frac{\alpha_{\overline{\mathrm{MS}}}(\mu)}{4\pi} + \mathcal{O}(\alpha^2)\right] \quad (14)$$

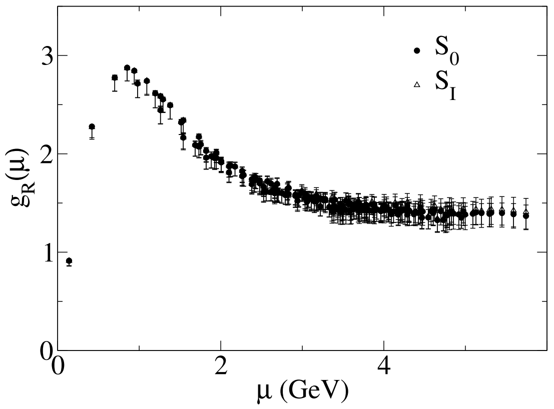

Figure 2. The running coupling $g_{\widetilde{\mathrm{MOM}}}(\mu)$ as a function of the renormalisation scale μ. The symbols are as in fig. 1.

which gives $\Lambda_{\widetilde{\mathrm{MOM}}}/\Lambda_{\overline{\mathrm{MS}}} = \exp(151/232) = 1.77$. From fig. 3 we find $\Lambda_{\widetilde{\mathrm{MOM}}} = 300 \pm 100$ MeV, which gives an estimate of $\Lambda_{\overline{\mathrm{MS}}} = 170 \pm 65$ MeV. As already indicated, however, the systematic uncertainties connected with the finite lattice spacing (even using the S_I, giving improved ultraviolet behaviour) and the power corrections to α_s (which come in addition to those arising in eq. (13)) are substantial, and may easily change this estimate by a factor of 2.

4. OUTLOOK

We have defined a zero-momentum ($\widetilde{\mathrm{MOM}}$) subtraction scheme for the quark–gluon vertex and used it to determine α_s and Λ_{QCD}. Lattice artefacts still give substantial uncertainties; it is not clear whether they are under control. A further source of systematic error in the determination of Λ_{QCD} is power corrections to α_s. Work is in progress to determine these.

In the Landau gauge, longitudinal components of the vertex can only be studied at zero gluon momentum, so $\widetilde{\mathrm{MOM}}$ is the only feasible renormalisation scheme. Transverse components, which are all zero at this point, may be studied in more general kinematics. We are currently analysing the two components $\lambda_3(p^2, 0, p^2)$ and $\tau_3(p^2, 4p^2, p^2)$.

In a general covariant gauge, in addition to studying the gauge dependence of the vertex, it is also possible to define a symmetric ($\overline{\mathrm{MOM}}$) renormalisation scheme. This is an interesting issue for future work.

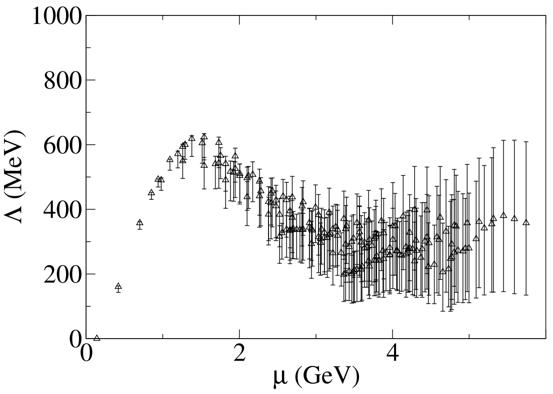

Figure 3. $\Lambda_{\widetilde{\mathrm{MOM}}}$ evaluated using (11) as function of the scale μ, using the 'improved' propagator S_I.

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