THE NONPERTURBATIVE QUARK–GLUON VERTEX*

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We show results for the quark–gluon vertex in the Landau gauge, using a mean-field improved Sheikholeslami–Wohlert fermion action. We compute all the three non-zero form factors of the vertex at zero gluon momentum, and compare them to the abelian vertex. The quark mass dependence of the vertex is also investigated and found to be negligible for the range of masses considered.

1. Introduction

The quark–gluon vertex plays an important role in the dynamics of QCD. A non-trivial infrared structure of the vertex can affect hadronic observables through the running of the relevant coupling, while at a more fundamental level it is thought crucial to achieve confinement and a sufficient degree of dynamical chiral symmetry breaking. Therefore, a first-principles, non-perturbative determination of the vertex, which the lattice may provide, is highly desirable.

In a previous paper[1], results for the form factor (λ_1) proportional to the running coupling were presented. Here, we extend this by also studying the two other nonzero form factors at zero gluon momentum. We also study the mass dependence of these form factors.

We use the notation and the decomposition of the vertex given in Ref[1]. The outgoing quark momentum is denoted p and the (outgoing) gluon mo-

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mentum q. The incoming quark momentum is thus $k = p + q$. For $q = 0$, the vertex can be written

$$\Lambda_\mu(p,q) = -ig\Big[\lambda_1(p^2)\gamma_\mu - 4\lambda_2(p^2)\,\not{p}p_\mu - 2i\lambda_3(p^2)p_\mu\Big]. \tag{1}$$

In an abelian theory (QED), the Ward–Takahashi identities mean that these form factors are given uniquely in terms of the fermion propagator $S^{-1}(p) = i\,\not{p}A(p^2) + B(p^2)$ by[2]

$$\lambda_1^{\mathrm{QED}}(p^2) = A(p^2) \qquad \lambda_2^{\mathrm{QED}} = -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}p^2}A(p^2) \qquad \lambda_3^{\mathrm{QED}} = -\frac{\mathrm{d}}{\mathrm{d}p^2}B(p^2) \tag{2}$$

By comparing with these expressions, we can thus get a direct measure of the non-abelian nature of the vertex.

All the results presented here are obtained in the Landau gauge, in the quenched approximation at $\beta = 6.0$, using the Sheikholeslami–Wohlert fermion action with mean-field improvement coefficients. We have used two quark masses, $\kappa = 0.137$ and 0.1381, corresponding to a bare quark mass $m = 118$ and 61 MeV respectively, with about 500 configurations in both cases. For further details about the calculation, we refer to Ref[1].

2. Results

In Figure 1 we show λ_1 at $q = 0$ for our two quark masses. We see a clear enhancement of the form factor in the infrared, and virtually no dependence on the quark mass beyond the three most infrared points. In the right-hand panel we show the deviation of the non-abelian form factor from the abelian expression in (1). Although much of the infrared enhancement is seen to be present in the abelian case, we also find a definite deviation of 50% for the most infrared points. However, the slight mass dependence in the infrared can be fully accounted for by the abelian behaviour.

In Figure 2, we show the form factors $p^2\lambda_2$ and λ_3 as functions of p, where we have subtracted off the lattice tree-level behaviour to bring them closer to the continuum. At large momenta this procedure implies large cancellations, and our results therefore contain little information beyond $p \sim 3$ GeV. The abelian form (1) has been determined by fitting the quark propagator form factors $Z = 1/A$ and $M = B/A$ to functional forms[3]. Our results for λ_2 appear to be quite consistent with the abelian form, except for the most infrared point where finite volume errors may be substantial. For λ_3, our results match the abelian form almost perfectly, indicating that the non-abelian contribution to this form factor may be negligible. In both cases, we find again at most a slight mass dependence.

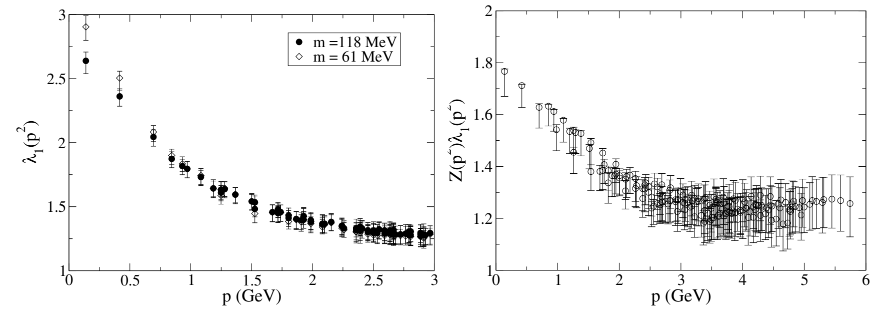

Figure 1. Left: The unrenormalised form factor $\lambda_1(p^2)$ as a function of p. Right: $\lambda_1(p^2)$ multiplied by the quark renormalisation function $Z(p)$, for $m = 118$ MeV. In an abelian theory, this would be a constant.

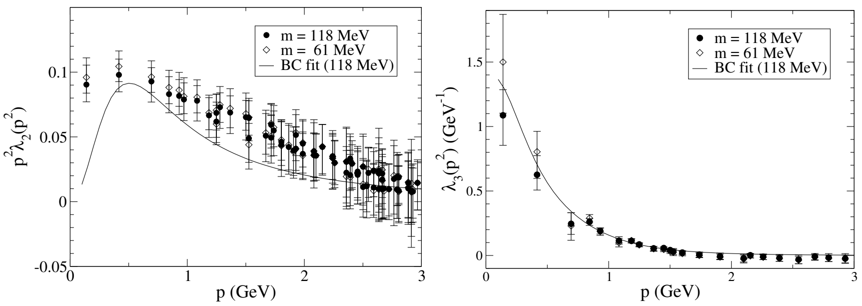

Figure 2. The unrenormalised form factors $p^2\lambda_2(p^2)$ (left) and $\lambda_3(p^2)$ (right) as functions of p. Also shown are the abelian (Ball–Chiu) forms of (1), for the heavier mass.

3. Outlook

In addition to the form factors presented here, we are currently studying the chromomagnetic moment form factor (τ_5 in the notation of Ref.[1]) at $q = -2p$. These results will be presented in a forthcoming paper. We are also planning to extend our study to the full kinematical space available. In the longer term, simulations using fermion actions with improved tree-level behaviour, and on larger volumes, will be performed in order to reduce systematic errors.

References

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