442 Tutorial Sheet 8[1]

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1. (3) Consider a simplified model of the history of a flat universe involving a period of inflation. The history is split into four periods: (a) $0 < t < t_3$ radiation only; (b) $t_3 < t < t_2$ vacuum energy dominates with an effective cosmological constant $\Lambda = 3t_3^2/4$; (c) $t_2 < t < t_1$ a period of radiation domination; (d) $t_1 < t < t_0$ matter domination.

   (a) Show that in (c) $\rho(t) = \rho_r(t)3\pi t^2/32$ and in (d) $\rho(t) = \rho_m(t) = \pi t^2/6$. The functions ρ_r and ρ_m are introduced for later convenience.

   (b) Give simple analytic formulas for $a(t)$ which are approximately true in these four epochs.

   (c) Shat that during the inflationary epoch the universe expands by a factor

   $$\frac{a(t_2)}{a(t_3)} = \exp\left(\frac{t_2 - t_3}{2t_3}\right) \tag{1}$$

   (d) In the notation introduced earlier, show

   $$\frac{\rho_r(t_0)}{\rho_m(t_0)} = \frac{9}{16}\left(\frac{t_1}{t_0}\right)^{2/3} \tag{2}$$

   (e) If $t_3 = 10^{-35}$ seconds, $t_2 = 10^{-32}$ seconds, $t_1 = 10^4$ years and $t_0 = 10^10$ years, give a sketch of $\log a$ against $\log t$ marking any important epochs.

   (f) Define what is meant by the particle horizon and calculate how it behaves for this model. Indicate this behavior on the sketch you made. How does inflation solve the horizon problem?

2. (4) After applying the harmonic gauge there is a residual invariance under coördinate transformation. For a gravitational wave in the z-direction, show that the polarization tensors can be taken to be e_+ and $e_\times$ where $e_+ = \text{diag}(0, 1, -1, 0)$ and $(e_\times)_{12} = (e_\times)_{21} = 1$ and all others zero. This was sketched in class, the challenge here is to do it more completely.

3. (2) By changing coördinates, describe the effect of an $e_\times$ polarized gravity wave.

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