442 Tutorial Sheet 4[1]

24 January 2005

1. (2) This question is revision of Lagrangian mechanics. Derive the Euler-Lagrance equations from $\delta S = 0$ where
$$S = \int L(t, \mathbf{q}, \dot{\mathbf{q}})dt \tag{1}$$

2. (2) Derive the Euler-Largrange equation
$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \tag{2}$$
from $\delta S = 0$ where
$$S = \int \mathcal{L}(t, \phi, \partial_\mu \phi) d^4x \tag{3}$$

3. (3) Derive the Euler-Largange equation
$$\nabla_\mu \frac{\partial \mathcal{L}}{\partial(\nabla_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \tag{4}$$
from $\delta S = 0$ where
$$S = \int \mathcal{L}(t, \phi, \nabla_\mu \phi)\sqrt{g} d^4x \tag{5}$$
Recall that $\nabla_\mu \phi = \partial_\mu \phi$ and that $\sqrt{g}\nabla_\mu X^\mu = \partial_\mu \sqrt{g} X^\mu$.

4. (4) Calculate the energy-momentum tensor
$$T_{\mu\nu} = F_{\lambda\mu} F^{\lambda}{}_{\nu} - \frac{1}{4} g_{\mu\nu} F_{\lambda\rho} F^{\lambda\rho} \tag{6}$$
for the Maxwell field:
$$S = -\frac{1}{4} \int F^{\mu\nu} F_{\mu\nu} \sqrt{g} d^4x \tag{7}$$
where
$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \tag{8}$$

[1] Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/442.html