442 Tutorial Sheet 3[1]

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1. (3) What is the curvature on a cylinder?

2. (4) The Weyl tensor is described as being the trace-free part of the Riemann tensor. It is a (0,4) tensor with the same symmetries as the Riemann tensor, linear in the Riemann tensor, having no dependence on derivatives of the metric except through the Riemann tenson and with all traces vanishing. Find a formula for the Weyl tensor in terms of the Riemann tensor, the Ricci tensor and the Ricci scalar. [The easiest way to do this is to write the most general expression with the correct symmetries and the determine any arbitrary constants by contracting and using the defining quality of the Weyl tensor: it has zero traces].

3. (3) Find the time-like geodesics for the metric

$$ds^2 = \frac{1}{t^2}\left(-dt^2 + dx^2\right) \tag{1}$$

You might want to use the integral

$$\int \frac{dt}{t\sqrt{1+C^2t^2}} = \frac{1}{2}\log\left(\frac{\sqrt{1+C^2t^2}-1}{\sqrt{1+C^2t^2}+1}\right) \tag{2}$$

[Rather than writting out the geodesic equations, it may be easier to note that x is ignorable and then use proper time as the parameter. This resulting equation is an integral of the geodesic equation and can be solved to give $t(\tau)$ and $x(\tau)$.]

4. (2) Poisson's fomulation of Newtonian gravity is

$$\begin{aligned} \nabla^2\phi &= 4\pi\rho \\ \mathbf{g} &= -\nabla\phi \end{aligned} \tag{3}$$

where ρ is the matter density, ϕ is the gravitational potential and g is the accelleration due to gravity. Show this gives the usual Newtonian formula for a point-like source.

5. (1) What is the value of ϕ on the surface of the Earth.

6. (3) Find the Newtonian limit of Einstein gravity with cosmological constant.

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