442 Tutorial Sheet 1[1]

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1. (2) Find an expression for
$$\nabla_c\nabla_d T^a{}_b - \nabla_d\nabla_c T^a{}_b \tag{1}$$
in terms of the Riemann tensor.

2. (2) Calculate the usual metric on the surface of a sphere by considering a radius r sphere $x^2 + y^2 + z^2 = r^2$ embedded in three-dimensional flat space $ds^2 = dx^2 + dy^2 + dz^2$. To do this, change to spherical polar coördinates:
$$\begin{aligned} x &= r\cos\phi\sin\theta \\ y &= r\sin\phi\sin\theta \\ z &= r\cos\theta \end{aligned} \tag{2}$$
and then set $dr = 0$ to restrict to surface of the sphere.

3. (4) Find the curvature on a two-dimensional hyperboloid:
$$t^2 - x^2 - y^2 = r^2 \tag{3}$$
embedded in Minkowski space:
$$ds^2 = -dt^2 + dx^2 + dy^2 \tag{4}$$
In other words, change to hyperbolic coördinates
$$\begin{aligned} x &= r\cos\phi\sinh\eta \\ y &= r\sin\phi\sinh\eta \\ t &= r\cosh\eta \end{aligned} \tag{5}$$
and then restrict to the surface of the hyperboloid by setting $dr = 0$. This gives the metric on the surface of the embedded hyperboloid, now calculate its Ricci scalar.

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