123 Problem Sheet Hillary 1[1]

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1. Use L'Hôpidal's rule to calculate

$$\lim_{x\to 1}\frac{x\log_e x}{1-x} \tag{1}$$

$$\lim_{x\to\pi/2}\frac{\sin x-1}{(x-\pi/2)^2} \tag{2}$$

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{\sqrt{1-x}} \tag{3}$$

$$\lim_{x\to 0}\frac{\log_e\cos x}{\sin^2 x} \tag{4}$$

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{1-x}. \tag{5}$$

If these limits are taking you a long time, then leave them after doing one or two and go on to the Taylor series questions.

2. Find the first three terms of the Taylor expansion of $\tan x$ around $x=0$.

3. Find the first three term of the Taylor expansion of $1/x$ around $x=1$.

4. Expand

$$\frac{1}{\sqrt{1+h}}$$

for small h. Include terms of order two in h but stop there.

5. Expand $(2+x)^{1/3}$ near $x=2$. What is the third order term.

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