123 Problem Sheet Hillary 1[1]
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1. Use L'Hôpidal's rule to calculate

$$\lim_{x \to 1} \frac{x \log_e x}{1 - x} \tag{1}$$

$$\lim_{x \to \pi/2} \frac{\sin x - 1}{(x - \pi/2)^2} \tag{2}$$

$$\lim_{x \to 1} \frac{1 - \sqrt{x}}{\sqrt{1 - x}} \tag{3}$$

$$\lim_{x \to 0} \frac{\log_e \cos x}{\sin^2 x} \tag{4}$$

$$\lim_{x \to 1} \frac{\sqrt{x + 3} - 2}{1 - x}. \tag{5}$$

If these limits are taking you a long time, then leave them after doing one or two and go on to the Taylor series questions.

2. Find the first three terms of the Taylor expansion of $\tan x$ around $x = 0$.

3. Find the first three term of the Taylor expansion of $1/x$ around $x = 1$.

4. Expand

$$\frac{1}{\sqrt{1 + h}}$$

for small h. Include terms of order two in h but stop there.

5. Expand $(2 + x)^{1/3}$ near $x = 2$. What is the third order term.

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