MA3466 Tutorial Sheet 4, outline solutions, except q2[1]

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2 Consider the game of $\&^{\mathrm{TM}}$: a board game with numbered squares, the idea being to race your token along the square by flipping a coin, heads you advance one place, tails you stay put. Let Y_1, Y_2 and Y_3 be you position after one, two and three turns. Clearly $Y_1 \to Y_2 \to Y_3$, show this by calculating the conditional probabilites and showing

$$p(x,y,z) = p(x)p(x|y)p(z|y) \tag{1}$$

Calculate $I(Y_1;Y_2)$ and $I(Y_1;Y_3)$.

*Solution:*Well $p(x,y,z)$ has eight elements but they are all the same since the actual coin flips are independent, hence

$$\begin{aligned} p(0,0,0) &= p(0,0,1) = p(0,1,1) = p(0,1,2) \\ &= p(1,1,1) = p(1,1,2) = p(1,2,2) = p(1,2,3) = \frac{1}{8} \end{aligned} \tag{2}$$

Now the thing is the marginal distributions aren't so evenly distributioned, just looking above

$$p_{Y_3}(0) = \frac{1}{8},\ p_{Y_3}(1) = \frac{3}{8},\ p_{Y_3}(2) = \frac{3}{8},\ p_{Y_3}(1) = \frac{1}{8} \tag{3}$$

for example. However, the marginal distributions are also all the same; given Y_2 for example, there are two possilibities for Y_3, it can be the same as Y_2 for tails and one greater than Y_2 for heads, so

$$p_{Y_2|Y_3}(y,z) = \frac{1}{2} \tag{4}$$

and hence

$$p(x,y,z) = p(x)p(x|y)p(z|y) \tag{5}$$

simply reduces to

$$\frac{1}{8} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \tag{6}$$

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