MA3466 Tutorial Sheet 7[1]

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1. (C& T 5.2) Let $p(S_i) = p_i$ for some set of outcomes $\{S_1, S_2, \ldots, S_n\}$. The S_i's are uniquely encoded into strings from a D-symbol alphabet in a uniquely decodable manner. If $n = 6$ and the code word lengths are $(l_1, l_2, \ldots, l_n) = (1, 1, 2, 3, 2, 3)$ find a good lower bound on D.

2. (C& T 5.4) Slackness in the Kraft inequality. An instantaneous code has word lengths l_1 to l_m satisfying the strict inequality
$$\sum_{i=1}^{m} D^{-l_i} < 1 \tag{1}$$
Show there are arbitrarily long sequences of code symbols in $\mathcal{D}^*$ which cannot be decoded into sequences of codewords: that is, not all sequences of symbols in D form a sentence.

3. Work out Huffman codes for

   (a) $p(A) = .5$, $p(B) = .2$, $p(C) = .1$, $p(D) = .1$ and $p(E) = .1$ with $D = 2$.

   (b) $p(A) = .25$, $p(B) = .15$, $p(C) = .1$, $p(D) = .1$, $p(E) = .1$, $p(F) = .1$, $p(G) = .1$, $p(H) = .05$ and $p(I) = .05$ with $D = 2$, $D = 3$ and $D = 4$.

   (c) $p(A) = .15$, $p(B) = .15$, $p(C) = .15$, $p(D) = .1$, $p(E) = .1$, $p(F) = .1$, $p(G) = .1$, $p(H) = .1$ and $p(I) = .05$ with $D = 2$ and $D = 5$.

   Work out the average code length and the entropy in each case.

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