MA3364 Tutorial Sheet 6[1]

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1. (C& T 3.2) AEP and mutual information. Let(X_i, Y_i) be i.i.d with joint distributions $p(x, y)$. We form the log likelihood ration of the hypothesis that X and Y are independent versus the hypothesis that they are dependent. What is the limit of

$$\frac{1}{n} \log \frac{p(\mathbf{X})p(\mathbf{Y})}{p(\mathbf{X}, \mathbf{Y})} \tag{1}$$

2. (C& T 3.3) A piece of cake. A cake is sliced roughly in half and the largest piece selected each time, the other bits being discarded. Assume $p(2/3, 1/3) = 3/4$ and $p(2/5, 3/5) = 1/4$. How large, to the first order in the exponent, is the piece of cake after n cuts.

3. (C& T 3.6) AEP-like limit. Let X_1, X_2 and so on be i.i.d., drawn with distribution $p(x)$, what is

$$\lim_{n\to\infty} [p(X_1, X_2, \ldots, X_n)]^{1/n} \tag{2}$$

   For this you need to know the strong law of large numbers: to prove the AEP we used the weak law:

$$\frac{1}{n} \sum X_i \to EX \tag{3}$$

   in probability, the strong law states that it approaches it almost surely.

4. (C& T 3.13) This requires a table, so you need to look at this problem in the book; however the table in the books has some errors, for example, for $k = 15$ the middle figure should be .1616, not .5754.

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