MA3466 Tutorial Sheet 3[1]

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1. (C&T 2.2) Entropy of functions. Let X be a random variable taking on a finite number of values. What is the general inequality relating $H(X)$ and $H(Y)$ if

   (a) $Y = 2^X$

   (b) $Y = \cos X$

2. (C&T 2.4) Entropy of functions of a random variable. Let X be a discrete random variable. Show that the entropy of a function of X is less than or equal to the entropy of X by justifying the following steps

$$\begin{aligned} H(X, g(X)) &= H(X) + H(g(X)|X) \\ &= H(X), \\ H(X, g(X)) &= H(g(X)) + H(X|g(X)) \geq H(g(X)) \end{aligned} \tag{1}$$

   and hence $H(g(X)) \leq H(X)$.

3. (C&T 2.8) Drawing with and without replacement. An urn contains r red, w white and b black balls. Which has higher entropy, drawing $k \geq 2$ balls from the urn with replacement or without replacement?

4. (C&T 2.14) Enropy of a sum. Let X and Y be random variables that take on values x_1, x_2, ..., x_r and y_1, y_2, ..., y_s respectively. Let $Z = X + Y$.

   (a) Show that $H(Z|X) = H(Y|X)$. Argue that if X and Y are independent then $H(Y) \leq H(Z)$ and $H(X) \leq H(Z)$. Thus the addition of independent random variables add uncertainy.

   (b) Give an example of random variables for which $H(X) > H(Z)$ and $H(Y) > H(Z)$.

   (c) Under what conditions does $H(Z) = H(X) + H(Y)$.

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