MA3466 Tutorial Sheet 2[1]

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1. (C&T 2.6) Find joint random variables X, Y and Z such that

   (a) $I(X;Y|Z) < I(X;Y)$

   (b) $I(X;Y|Z) > I(X;Y)$

2. (C&T 2.7) Suppose that one has n coins, among which there may or may not be one counterfeit coin. If there is a conterfeit coin it will weight either less or more than the other coins. The coins are weighed using a balance.

   (a) Find an upper bound on the number of coins n so that k weighings will find the couinterfeit coin, if any, and correctly declare it to be heavier or lighter.

   (b) What is the coin-weighing strategy for $k = 3$ weighings and 12 coins/

3. (C&T 2.9) Let X_1 and X_2 be discrete random variables drawn accorind to distributions p_1 and p_2 from their respective alphabets $\mathcal{X}_1 = \{1, 2, \ldots, m\}$ and $\mathcal{X}_2 = \{m+1, m+2, \ldots, n\}$. Let

$$X = \begin{cases} X_1 & \text{with probability } \alpha \\ X_2 & \text{with probability } 1-\alpha \end{cases} \tag{1}$$

   (a) Find $H(X)$ in terms of $H(X_1)$ and $H(X_2)$.

   (b) Maximize over α to show that

$$2^{H(X)} \leq 2^{H(X_1)} + 2^{H(X_2)} \tag{2}$$

4. (C&T 2.12). Let $p(x, y)$ be given by $p(0, 0) = p(0, 1) = p(1, 1) = 1/3$ and $p(1, 0) = 0$. Find $H(X)$, $H(Y)$, $H(X|Y)$, $H(Y|X)$, $H(X, Y)$, $H(Y) - H(Y|X)$ and $I(X;Y)$.

5. Prove the equals part of Jensen's inequality: if f is stricly cup-like on an interval which includes all outcomes

$$\langle f(X)\rangle = f(\langle X\rangle) \tag{3}$$

   if and only if $X = \langle X\rangle$ with probability one.

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