**MA3466 Tutorial Sheet 1**[1]

29 January 2010

1. For two random variables with numerical outcomes, find $p(x, y)$ so that there is zero correllation
$$\langle x'y' \rangle = 0 \tag{1}$$
but $X$ and $Y$ aren't independent. $x' = x - EX$ and $y' = y - EY$.

2. Work out the marginal distributions and the $x = a$ conditional distribution for

| | $a$ | $b$ |
|---|---|---|
| 1 | $\frac{1}{3}$ | $\frac{1}{6}$ |
| 2 | 0 | $\frac{1}{4}$ |
| 3 | $\frac{1}{8}$ | $\frac{1}{8}$ |

3. (C&T 2.1) A fair coin is flipped until the first head occurs. Let $X$ denote the number of flips required.

   (a) Find the entropy $H(X)$ in bits. The following expressions may be useful:
$$\begin{aligned}\sum_{n=0}^{\infty} r^n &= \frac{1}{1-r} \\ \sum_{n=0}^{\infty} nr^n &= \frac{r}{(1-r)^2}\end{aligned} \tag{2}$$

   (b) A random variable $X$ is drawn according to this distribution. Find an efficient sequence of yes-no questions of the form, 'Is $X$ contained in the set $S$?'. Compare $H(X)$ to the expected number of questions required to determine $X$.

4. (C&T 2.3) What is the minimum value of $H(p_1, p_2, \ldots, p_n) = H(\mathbf{p})$ as $\mathbf{p}$ ranges overall possible vectors. Find the $\mathbf{p}$ which achieve this bound. $H(p_1, p_2, \ldots, p_n)$ is a common notation for $H(X)$ where $X$ has $n$-outcomes $\{x_1, x_2, \ldots, x_n\}$ and $p_1 = p(x_1)$, $p_2 = p(x_2)$ and so on.

[1] Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/MA3466`