MA22S3 Tutorial Sheet 2, outline solutions.[1][2]

20 October 2010

Questions

1. (2) What is the period of $\sin t\cos t$ and $\sin^3 t$.

   *Solution:*Since $\sin t$ has period 2π, so does $\sin^3 t$, however $\sin t\cos t$ has period π since it is equal $\sin 2t/2$.

2. (2) Find the Fourier series for $\sin^3 t$; a quick way to do this is to regard it as a trigonometry problem, rather than a Fourier series problem, that is use the trigonometric identities to express it in terms of sines and cosines, rather than doing all the integrals: so start by writing $\sin^3 t = \sin^2 t\sin t$ and then write $\sin^2 t$ in terms of $\cos 2t$.

   *Solution:*Using trig identities

   $$\sin^3 t = \sin^2 t\sin t = \frac{1}{2}(1-\cos 2t)\sin t \tag{1}$$

   and again use the trig identity for $\cos 2t\sin t$ to give

   $$\sin^3 t = -\frac{1}{4}\sin 3t + \frac{3}{4}\sin t \tag{2}$$

   In other words, $b_1 = 3/4$ and $b_3 = -1/4$ and all the others are zero.

3. (4) Express the following periodic function ($L = 2\pi$) as complex Fourier series

   $$f(t) = \begin{cases} 0 & -\pi < t < 0 \\ 1 & 0 < t < a \\ 0 & a < t < \pi \end{cases} \tag{3}$$

   where $a \in (0,\pi)$ is a constant.

   Solution:$f(x) = \sum_{n\in Z} c_n e^{int}$ with

   $$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} dt\ e^{-int} f(t) = \frac{1}{2\pi}\int_0^a dt\ e^{-int}$$

   so that $c_0 = a/2\pi$ and

   $$c_n = \frac{1}{2\pi}\left.\frac{e^{-int}}{-in}\right|_0^a = \frac{i}{2\pi n}\left(e^{-ian}-1\right).$$

[1]Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/MA22S3`

[2]Including material from Chris Ford, to whom many thanks.