MA22S3 Tutorial Sheet 2, outline solutions.¹²

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Questions

1. (2) What is the period of $\sin t \cos t$ and $\sin^3 t$.

Solution: Since $\sin t$ has period 2π , so does $\sin^3 t$, however $\sin t \cos t$ has period π since it is equal $\sin 2t/2$.

2. (2) Find the Fourier series for $\sin^3 t$; a quick way to do this is to regard it as a trigonometry problem, rather than a Fourier series problem, that is use the trigonometric identities to express it in terms of sines and cosines, rather than doing all the integrals: so start by writing $\sin^3 t = \sin^2 t \sin t$ and then write $\sin^2 t$ in terms of $\cos 2t$.

Solution: Using trig identities

$$\sin^3 t = \sin^2 t \sin t = \frac{1}{2} (1 - \cos 2t) \sin t \tag{1}$$

and again use the trig identity for $\cos 2t \sin t$ to give

$$\sin^3 t = -\frac{1}{4}\sin 3t + \frac{3}{4}\sin t \tag{2}$$

In other words, $b_1 = 3/4$ and $b_3 = -1/4$ and all the others are zero.

3. (4) Express the following periodic function $(L = 2\pi)$ as complex Fourier series

$$f(t) = \begin{cases} 0 & -\pi < t < 0\\ 1 & 0 < t < a\\ 0 & a < t < \pi \end{cases}$$
(3)

where $a \in (0, \pi)$ is a constant.

Solution: $f(x) = \sum_{n \in \mathbb{Z}} c_n e^{int}$ with

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} dt \ e^{-int} f(t) = \frac{1}{2\pi} \int_{0}^{a} dt \ e^{-int}$$

so that $c_0 = a/2\pi$ and

$$c_n = \frac{1}{2\pi} \left. \frac{e^{-int}}{-in} \right|_0^a = \frac{i}{2\pi n} \left(e^{-ian} - 1 \right).$$

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