MA22S3 ODE work sheet.¹

16 February 2010

Questions: lots of ODE questions for practising.

- 1. Find the general solution for
 - (a) $t\dot{y} + y = t^4$
 - (b) $(1 t^2)\dot{y} ty = 1$
 - (c) $(t-2)\dot{y} y = (t-2)^3$

In the third example, what is the solution if y = 10 when t = 4.

2. Bernoulli Equations: a Bernoulli equation is an equation of the form

$$\dot{y} + py = qy^n \tag{1}$$

where p and q are functions of t. By dividing both sides by y^n and rewriting in terms of $z = y^{1-n}$ reduce the Bernoulli equation to a first order linear inhomogeneous differential equation for z.

3. Find the general solution for

(a)
$$\ddot{y} + 5\dot{y} + 6y = 0$$

(b)
$$\ddot{y} + 4\dot{y} - 21y = 0$$

(c) $\ddot{y} + 11\dot{y} + 18y = 0$

In each example, what is the solution if y(0) = 2 and $\dot{y}(0) = -3$.

- 4. Find the general solution for
 - (a) $\ddot{y} + 4\dot{y} + 4y = 0$
 - (b) $\ddot{y} + 10\dot{y} + 25y = 0$
 - (c) $\ddot{y} + 8\dot{y} + 16y = 0$

In each example, what is the solution if y(0) = 2 and $\dot{y}(0) = -3$.

- 5. Find the general solution for $\ddot{y} 2\dot{y} + 10y = 0$.
- 6. Find the general solution for $\ddot{y} 5\dot{y} + 6y = 25$, what is the solution is y(0) = 4 and $\dot{y}(0) = 1$.

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- 7. Find the general solution for $\ddot{y} 5\dot{y} + 6y = e^{-3t}$, what is the solution is y(0) = 4 and $\dot{y}(0) = 1$.
- 8. Find the general solution for $\ddot{y} 5\dot{y} + 6y = e^{3t}$, what is the solution is y(0) = 4 and $\dot{y}(0) = 1$.
- 9. Find the general solution for $\ddot{y} 6\dot{y} + 9y = e^{3t}$, what is the solution is y(0) = 4 and $\dot{y}(0) = 1$.
- 10. Find the general solution for $\ddot{y} 5\dot{y} + 6y = 2\sin 4t$, what is the solution is y(0) = 4and $\dot{y}(0) = 1$.
- 11. Find the general solution for $\ddot{y} + 6\dot{y} + 10y = 2\sin 2t$.