MA22S3 Fourier work sheet.[1]

16 November 2010

Questions: some reasonably hard questions for practising your Fourier analysis.

1. What is the Fourier series for $f(t) = t^2$ for $t \in (-\pi, \pi)$ and $f(t + 2\pi) = f(t)$?

2. A periodic function $f(t)$ with period two is defined by

$$f(t) = \begin{cases} 3t & 0 < t < 1 \\ 3 & 1 < t < 2 \end{cases} \tag{1}$$

   Sketch the function and calculate its Fourier series

3. Using the complex Fourier series of $f(t) = 2t/T$ on $0 < t < T$ and $f(t + 2T) = f(t)$ and Parseval's theorem, show

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \tag{2}$$

4. Calculate the complex Fourier series of the square pulse function, $f(t + 2T) = f(t)$ and

$$f(t) = \begin{cases} 0 & -T < t < -d \\ A & -d < t < d \\ 0 & d < t < T \end{cases} \tag{3}$$

5. Derive the formula for c_n directly from the corresponding formulae for a_n and b_n.

6. Calculate the Fourier transform of the on-off pulse

$$f(t) = \begin{cases} -A & -d < t < 0 \\ A & 0 < t < d \\ 0 & |t| > T \end{cases} \tag{4}$$

7. Define the Fourier cosine transform as $F_c(x) = \int_0^\infty f(t) \cos xt dt$ and show the function $f(t) = \cos(at)$ for $0 \geq t \geq a$ and zero otherwise, has

$$F_c(x) = \frac{1}{2}\left[\frac{\sin(1+x)a}{1+x} + \frac{\sin(1-x)a}{1-x}\right] \tag{5}$$

8. The Fourier sine transform is $F_s(x) = \int_0^\infty f(t) \sin xt dt$; what are the Fourier sine and cosine transforms of

$$f(t) = \begin{cases} 1 & 0 \leq t \leq a \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

9. If $\widetilde{f(k)}$ is the Fourier transform of $f(t)$ what is the Fourier transform of $f(t - a)$ where a is a constant.

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