MA22S3 Tutorial Sheet 8.[1,2]

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Useful facts:

- Euler's eqn: use $t = \exp z$ to change
$$\alpha t^2\ddot{y} + \beta\dot{y} + \gamma y = 0 \tag{1}$$
to
$$\alpha\frac{d^2y}{dz^2} + (\beta - \alpha)\frac{dy}{dz} + \gamma y = 0 \tag{2}$$
and then solve that as an example of a damped harmonic oscillator.

- Remember $e^{ab} = (e^a)^b$.

- Series solution: assume there is a solution of the form
$$y = \sum_{n=0}^{\infty} a_n t^n$$
and, by substituting into the equation find a recursion relation: an equation relating higher terms in a_n to lower one.

- By expanding out the sum it is easy to see $y(0) = a_0$ and $\dot{y}(0) = a_1$

Questions

1. (2) Solve $t^2\ddot{y} + 4t\dot{y} + 2y = 0$.

2. (2) Solve $x^2\ddot{y}+4t\dot{y}+2y = t^5$. In this example there is a forcing term, but substituting for t^5 in terms of z will just give a forced damped harmonic oscillator which can be solved then in the usual way.

3. (4) Assuming the solution of
$$(1 - t)\dot{y} + y = 0 \tag{3}$$
has a series expansion about $t = 0$ work out the recursion relation. Write out the first few terms and notice that the series $a_2 = 0$ so the series actually terminates to give $y = a_0(1 - t)$. What is the solution with $y(0) = 2$.

[1] Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/231
[2] Including material from Chris Ford, to whom many thanks.