MA22S3 Tutorial Sheet 7.[1]

9 December 2010

Useful facts:

- To solve the equation $a\ddot{y} + b\dot{y} + cy = 0$, with a, b and c constants, use an exponential substitution $y = \exp(\lambda t)$ to get the auxilary equation $a\lambda^2 + b\lambda + c = 0$ and solve for λ. Usually this will give two solutions.
- If there is only one λ then $y = t\exp(\lambda t)$ will also be a soln.
- The solution of the equation $a\ddot{y}+b\dot{y}+cy = f(t)$ is $y = y_c+y_p$ where $y_c = C_1y_1+C_2y_2$ is the solution of $a\ddot{y} + b\dot{y} + cy = 0$.
- If $f(t) = \exp(\mu t)$ and μ isn't a solution of the auxillary equation, then substitute $y = C\exp(\mu t)$ and solve for C. If μ is a solution to the auxillary equation, use $y = Ct\exp(\mu t)$ or even $y = Ct^2\exp(\mu t)$ if the auxillary equation has two repeated roots.
- If $f(t) = f_1(t) + f_2(t)$ then solve $a\ddot{y} + b\dot{y} + cy = f_1(t)$ and $a\ddot{y} + b\dot{y} + cy = f_2(t)$ and add the solutions.
- $\cosh t = [\exp(t) + \exp(-t)]/2$

Questions

1. (2) Obtain the general solution to
$$\ddot{y} + 8\dot{y} + 16y = 0 \tag{1}$$
2. (2) Obtain the general solution to
$$\ddot{y} + 16y = 0 \tag{2}$$
3. (2) Obtain the solution to
$$\ddot{y} + \dot{y} - 2y = e^{5t} \tag{3}$$
with $y(0) = \dot{y}(0) = 0$.
4. (2) Obtain the general solution to
$$\ddot{y} + 8\dot{y} + 16y = 4\cosh t \tag{4}$$

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